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EX/ FTBE/MATH/T/114/2019(OLD)

B.E. FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. FIRST YEAR FIRST SEMESTER EXAMINATION – 2019(OLD)

MATHEMATICS-I

Time: Three hours　　　　　　　　　　　　　　　　　　　　Full Marks: 100

Answer any **Ten** questions　　　　　　　　　　　　　　　　　　10×10

1. If by an orthogonal transformation without change of origin the expression $ax^2 + 2hxy + by^2$ becomes $a'x'^2 + 2h'x'y' + b'y'^2$, then prove that $a + b = a' + b'$ and $ab - h^2 = a'\, b' - \, h'^2$. 10
2. Prove that the transformation of rectangular axes which converts $\frac{X^2}{p} + \frac{Y^2}{q}$ into $ax^2 + 2hxy + by^2$ will convert $\frac{X^2}{p-\gamma} + \frac{Y^2}{q-\gamma}$ into $\frac{ax^2+2hxy+by^2-\gamma(ab-h^2)(x^2+y^2)}{1-(a+b)\gamma+(ab-h^2)\gamma^2}$. 10
3. If l and l' are the lengths of the segments of any focal chord of the parabola $y^2 = 4ax$, prove that $\frac{1}{l} + \frac{1}{l'} = \frac{1}{a}$. 10
4. If the origin be at one of the limiting points of a system of co-axial circles of which $x^2 + y^2 + 2gx + 2fy + c = 0$ is a member, show that the equation of the system of circles cutting them all orthogonally is $(x^2 + y^2)\,(g + \mu f) + c(x + \mu y) = 0$. Show that the other limiting point is $\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$. 10
5. Show that the straight lines whose d. cs. are given by $al + bm + cn = 0$, $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$. 10
6. Prove that the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represent a pair of planes, if $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, and also prove that the angle between the planes is $\tan^{-1}\frac{2(f^2+g^2+h^2-bc-ca-ab)^{\frac{1}{2}}}{a+b+c}$ 10
7. Show that the locus of a variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$ 10
8. Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0, x - 2y + 3z + 1 = 0$ is a great circle. 10
9. (a)If $y = \left[\log\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)\right]^2 + k\log\left(x + \sqrt{x^2 - a^2}\right)$ then find the value of $(x^2 - a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = ?$

   (b)If $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$, $f'(0) = a, f(0) = b, then\ find\ the\ value\ of\ f''(x)$, where y is independent of x. 5+5
10. If $x\cos\theta + y\sin\theta = p$, touch the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$, then find the value of $(a\cos\theta)^n + (b\sin\theta)^n = ?$ 10

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11. Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1, x=0$ and parallel to the line $\frac{x}{a}+\frac{z}{c}=1, y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if $2d$ is the S.D., Prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{d^2}$. 10

12. A plane passes through a fixed point (α, β, γ) and cuts the co-ordinate axes in A, B, C. Prove that the locus of the centre of the sphere $OABC$ is given by $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=2$. 10