B.E.T.C.E. Examination, 2019 (OLD)
(1ST YR, 1ST SEM)
MATHEMATICS
PAPER - II G

Full Marks : 100 **Time: Three hours**

Answer any question 1 and any six from the rest. $4 + 6 \times 16 = 100$

1. Solve

$$z^6 + 2 = 0.$$ 4

2. (a) Show that the necessary and sufficient condition for a vector function $\overrightarrow{F}(t)$ to have constant magnitude is

$$\overrightarrow{F}(t) \cdot \frac{d\overrightarrow{F}(t)}{dt} = 0$$ 8

(b) Find the directional derivative of a scalar point function

$$f(x, y, z) = xzy$$

in the direction of the vector $\overrightarrow{i} - 2\overrightarrow{j} + \overrightarrow{k}$ at $(1, 2, 1)$. 8

3. (a). Determine the region of the z-plane for which

$$|z - 1| + |z + 1| \leq 3.$$ 8

(b). If $f(z) = u + iv$ is an analytic function of $z = x + iy$, then show that u and v both are harmonic functions. 8

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4. (a). Find the analytic function $f(z) = u + iv$ of which the real part

$$u = e^x(x \cos y - y \sin x).$$ 8

(b). Show that the function

$$f(z) = |xy|^{\frac{1}{2}}$$

satisfies Cauchy-Riemann equations at the origin but $f'(0)$ does not exist. 8

5. (a) Define with examples of regular point, singular point, isolated singularity and removal singularity. 8

(b) Evaluate the residues of $f(z)$ where

$$f(z) = \frac{e^2}{z^2(z^2+9)} \quad at\ z = 0.$$ 8

6. Evaluate using Cauchy's Residue theorem
(a)

$$\int_C \frac{z^2}{(z-2)(z+3)} dz,$$ 8

where C is the circle $| z |= 4$
(b)

$$\int_C \frac{3z-4}{z(z-1)} dz,$$

where C is the circle $| z |= 2$, 8

7. (a). Evaluate

$$\oint_c \frac{\cosh(\pi z)}{z(1+z^2)} dz,$$ 8

where c is circle $|z| = 2$.

(b). Evaluate the residues of $f(z)$ where

$$f(z) = \frac{e^2}{z^2(z^2+9)} \quad at\ z = 0, \ -3i,\ 3i.$$ 8

8. Define conservative field. Show that $\vec{F} = (\sin y + z)\vec{i} + (x\cos y - z)\vec{j} + (x - y)\vec{k}$ is a conservative field and find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$. 16

9. (a) If $\vec{r} \times d\vec{r} = 0$, show that $\vec{r}$ is a constant vector. 8

(b) Define solenoidal vector. Find a so that the vector

$$\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$$

is solenoidal. 8