B. ETCE 1st year 2nd sem exam- 2019 (old)
Mathematics-IV G
Group-A
(Answer any five questions)
(Use separate answerscript for this group)

1. Use Cramer's rule to solve the system of equations : 5

$$\begin{aligned} x + 2y + z &= 5 \\ 2x + 2y + z &= 6 \\ x + 2y + 3z &= 9 \end{aligned}$$

(b) Let $S = \{(x, y, z) \in R^3 : 3x - 5y = 0\}$. Then what is the dimension of the space S? Find a basis of S. 5

2. (a) Justify with reason whether the following mappings from R^3 to R^3 are linear or not 5
(i) $T(x, y, z) = (2x - y, 3y - 4, 5x - 6y)$
(ii) $S(x, y, z) = (2x, 3y, 4z)$
(b) Let 5

$$A^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 0 & 2 \end{pmatrix}$$

Then find a matrix B such that $AB = A^2 - A$.

3. (a) Let $T : R^3 \longrightarrow R^3$ be a linear operator defined by 5

$$T(1,1,1) = (1,0,0), T(1,1,0) = (0,1,0), T(1,0,0) = (0,0,1)$$

Then find the matrix of T w.r.t the standard basis $\{(1,0,0), (0,1,0), (0,0,1)\}$ of R^3.
(b) Find the rank of the following matrix 5

$$A = \begin{pmatrix} 1 & 3 & -2 & 2 & 3 \\ 1 & 4 & -3 & 4 & 2 \\ 2 & 3 & -1 & -2 & 10 \\ 1 & -1 & 2 & -6 & 8 \end{pmatrix}$$

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4. (a) Verify Cayley-Hamilton's theorem for the matrix A where 5

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

(b) Find the algebraic and geometric multiplicities of the eigenvalues of the following matrix 5

$$\begin{pmatrix} -1 & 1 & 1 \\ -3 & 3 & 1 \\ -4 & 3 & 2 \end{pmatrix}$$

5. (a) Let 5

$$A = \begin{pmatrix} -3 & 2 & 2 \\ -6 & 5 & 2 \\ -7 & 4 & 4 \end{pmatrix}$$

Then find a non-singular matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix consisting of the eigenvalues of A.

(b) Find the minimal polynomial of the matrix

$$\begin{pmatrix} 2 & 6 & -1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5

6. (a) Let A and B be two $n \times n$ matrices. Show that AB and BA have the same eigenvalues. 5

(b) Compute AB using block multiplication where 5

$$A = \begin{pmatrix} 2 & -1 & . & 1 & 0 \\ 0 & 4 & . & 0 & 1 \\ . & . & . & . & . \\ 0 & 0 & . & 2 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 0 & . & 2 \\ 0 & 1 & . & 3 \\ . & . & . & . \\ 2 & 3 & . & -1 \\ 1 & 2 & . & 1 \end{pmatrix}$$

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Ref. No. EX/ET/MATH/T/124/

Bachelor of Engineering Examination
(Year, 2nd Semester)

MATHEMATICS IVG

Full Marks:100 Time: Three hours

GROUP B
All questions carry equal marks
Symbols/Notations have there usual meanings
Answer **any FIVE** questions

7. Consider the equation

$$\frac{dx}{dt} = 2y$$
$$\frac{dy}{dt} = -9x - 10y.$$

Discuss the nature and stability properties of the critical point of the above equation.

8. Determine the nature of the point $x = 0$ of the equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0.$$

Hence obtain the series solution of the above equation about $x = 0$ when α is neither zero nor an integer.

9. Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$

10. Solve

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = e^x$$

11. Find power series solution of

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \alpha(\alpha + 1)y = 0.$$

α being a constant.

12. Solve the following

$$i)\ \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$$

$$ii)\ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = xe^{x}$$

13. Solve

$$3\frac{dx}{dt} + \frac{dy}{dt} + x - 2y = 0$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x - 13y = 0$$