Determine which states are recurrent and which are transient.

10. a) If A and B are two independent events in a random experiment then show that following are independent.

    i) A^C and B,

    ii) A^C and B^C.

    b) A card is drawn from a full pack and replaced 260 times. Find the probability of obtaining queen of hearts 4 times.

11. A point P is taken at random on a line segment AB of length 2a. Find the probability that the area of the rectangle consisting of the sides AP and PB will exceed $\frac{1}{2}a^2$.

12. a) Write down Chapman - Kolmogorov equation. Hence derive its Forward and Backward equation.

    b) Find mean and variance of the Binomial (n, p) distribution where n and p respectively denote the number of trials and probability of success.

B. E. Electronics Tele-Communication Engineering Examination, 2019

(1st Year, 2nd Semester , Old)

Mathematics – IIIG

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use separate Answer - script for each Part

Unexplained Notations & Symbols have their usual meanings.

PART - I

Answer ***any five*** questions $10\times5=50$

1. a) List all the elements of the group $\mathbb{Z}_2 \times \mathbb{Z}_3$ and find their order.

   b) Prove that S_3 is not a cyclic group.

   c) Explain that $2\mathbb{Z}$ U $3\mathbb{Z}$ is not a subgroup of the group $(\mathbb{Z},+)$.

2. a) Prove that $P = \{2, 3, 4, ..., 10\}$ with divisibility relation is a poset. Draw its Hasse diagram. Explain that this poset is not a lattice.

   b) Prove that intersection of two subgroups of a group is a subgroup.

3. a) Define two different norms on the vector space $\mathbb{R}^2$.

   b) Give an example (with proper explanation) of a field with 4 elements.

[Turn over

4. a) Give an example (with proper explanation) of a noncyclic abelian group.

   b) Give an example (with proper explanation) of an infinite group such that each subgroup has finite number of cosets.

5. a) Let X be a nonempty set and P(X) be its power set. Prove that (P(X), +, .) is a Boolean only where '+' is symmetric difference and '.' is intersection.

   b) Find the projection of $(a, b, c) \in \mathbb{R}^3$ onto the subspace spanned by {(1, 0, 0), (0, 1, 0)}

   c) Define a Boolean Algebra and give an example.

6. a) Draw the Hasse diagram of the subgroup lattice of the group $\mathbb{Z}_2 \times \mathbb{Z}_2$. Explain that it is a nondistributive lattice.

   Also explain that it is a modular lattice.

   b) List all the proper subgroups of the group S_3. Identify which of these are normal.

PART - II

Answer ***any five*** questions 10×5=50

7. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the distribution of the number of success. Also find the mean and the variance of the number of success.

8. There are three identical urns containing white and black balls. The first urn contains 2 white and 3 black balls, the second urn 3 white and 5 black balls, and the third urn 5 white and 2 black balls. An urn is chosen at random and a ball is drawn from it. If the ball drawn is white, what is the probability that the second urn is chosen ?

9. Define Markov chain. Consider a Markov chain having the transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix}$$

[Turn over