Solution Of Reynolds Equation for Cylindrical Fluid Film Bearing by FDM Method

Thesis submitted in partial fulfillment of
the requirements for the degree of

Master of Mechanical Engineering

BY

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Date (Subrat Kumar Maharana)

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This thesis is solely dedicated to

my parents

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c	Radial clearance(m)
e	Eccentricity
h	Lubricant film thickness
L	Length of bearing (m)
D	Diameter of bearing (m)
p	Pressure in lubricant film
μ	Viscosity of lubricant medium(N s m^{-2})
ρ	Density of lubricant medium(kg m^{-3})
θ	Angular direction
N	Speed of journal in revolutions per minute(rpm)
ω	Angular speed (rad/s)
S	Somerfield Number

ABSTRACT

To study the stability of a rotor in a fluid film bearing, we represent the system as a spring mass damper system. As lubricant flows through the bearing, with increasing the speed of the journal pressure gets developed within the bearing which gives rise to a force which does not allow metal to metal contact and ensures smooth operation .To find the stiffness and damping coefficients the first step is to calculate the pressure distribution .The flow of lubricant in the fluid film bearing is governed by Reynolds equation which is a simplified form of Navier stokes equation. The analytical solution is available for short and long bearing approximations but for finite width bearing analytical solutions are found to be very difficult .In this work finite difference method is used to calculated to find the pressure distribution for a simple case and the work is validated with others as well as the analytical solutions..Using the pressure distribution the static equilibrium position of the journal in the bearing is calculated.

Chapter-1

INTRODUCTION

This chapter named as introduction provides a general introduction to the entire work. It gives an overall idea of fluid film bearing in the present work, objective and planning of the work and brief summary of the entire thesis sequentially.

1. Introduction

A rotor system is always supported by bearings that may be sliding or ball or roller bearing. To study the dynamic characteristics of the rotor system a general idea about the bearings is of utmost importance. In most of the heavy duty works the sliding bearings are used of which fluid film bearing is of the most important category.

When the rotor rotates in the fluid film bearing it is possible to model the rotor system as a spring mass-damper-system for simplification of analysis. The bearings influence the vibration of rotor system to a less or higher extent by their dynamic properties as compared to the other components of the assembly. It is observed that relatively elastic rotors have less influence on the bearing while the stiff rotor show more influence.

Finding the dynamics coefficients of the fluid film bearing has a very high importance in the field of rotor dynamics. The flow of lubricant in the fluid film bearing is governed by Reynolds equation given by O. Reynolds. For very short or very long bearings approximations the analytical solutions are readily available which are called Somerfield's solution .But for a real finite width bearing the analytical solutions are hardly possible so very less appreciable. So the numerical methods such as finite difference methods and finite elements take hold over analytical ones. For the design of a real bearing the solution of Reynolds equation is an inevitable step. After solving the same it is possible to find the load carrying capacity, friction coefficient, lubricant flow rate, static equilibrium positions and a lot more.

Many researchers in the last four to five decades have given heroic attempts to solve to solve Reynolds equation in various analytical and numerical methods. Here is an attempt to solve Reynolds equation for cylindrical fluid film bearings using finite difference method and to validate with other researchers. The solution of Reynolds equation is used to find load carrying capacity and the attitude angle. The eccentric position as well as the attitude angle gives the equilibrium position of the journal bearing.

1.1. Problem Formulation

The main objective of the project is to find the solution of Reynolds equation of cylindrical fluid film bearing using finite difference technique and finding the static equilibrium position of the journal in the bearing .The work is a validation of paper by .Wenjie Zhou, Xuesong Wei, Leqin Wang and Guangkuan [1]. They solved Reynolds equation by using Finite difference method and for determining the static equilibrium position 'two fold secant' method has been used.

The current work followed similar procedure but using Vogelpohl parameter and a after calculating the load bearing capacities by using trapezoidal rule a generalized iteration procedure using over relaxation was used to calculate the attitude angle. The eccentricity ratio when combined with the attitude angle gives the equilibrium position

1.2 Summary of the thesis

The thesis has been based on the work discussed above. It comprises five chapters.

The first chapter named as ‘Introduction’ gives a general introduction to the entire work. It gives a general idea of fluid film bearing, context of present work and objective and planning of work and brief summary of the entire thesis in a sequential manner.

The second chapter has been named as ***'Literature Review'*** where different works of lots of researchers up to the now have been surveyed, historical background has been explored.

The third chapter ‘Methodology’ gives the idea of mathematical equations and formulation and solution procedure has been explained.

In the fourth chapter ***'Results and discussion'*** the results of the work have been shown and discussed. The solution of the problem has been done by using Matlab software.

The fifth chapter '***conclusion and future scope of work***' shows the success and failure points of the present work and, gives motivation and directs the reader to future scope and modification.

Chapter-2

LITERATURE REVIEW

This chapter is titled as Literature Review, where different works of several researchers up to now are studied Historical background is emphasized and the theme of present work is planned.

2. *Literature Review*

Wenjie Zhou et al. [1] described that for finding the dynamic coefficients of a fluid film bearing, determination of static equilibrium position is a most important and inevitable step. It gives an idea of superliner iteration method for calculating the stable equilibrium position of the journal in finite length bearing. It showed how to solve the Reynolds equation using finite difference method .It gave an idea of over relaxation techniques for convergence and Simpson quadrature method to find the definite integrals. It gives a clear idea of boundary conditions for Reynolds equation. It gave the definition of equilibrium position and explains how different methods such as two fold secant method, secant methods and dichotomy method.

A.A. Raimondi and J Boyd [2] presented three papers in series which explains the iteration technique to solve the Reynolds equation for a finite width bearing .It takes into consideration coefficient of friction variable, side leakage and temperature rise etc. to give the solution of Reynolds equation. Though it could not explain properly the iteration techniques, still gives a basic picture of different variables .It give the solution in form of charts. Raimondi and Boyd were in fact the first members who made heroic attempts to solve finite width bearing using iteration technique.

J W Lund [3] presented the idea of representing the rotor system as spring mass damper system and how determination of dynamic coefficients of bearing gives a very crucial contribution in the field of rotor dynamics. It gives the idea of short and long bearing approximations and .It gives an idea of static equilibrium curve and defines it. It gives the generalized Reynolds equation, its boundary conditions and clear expression for film thickness and methods for computing dynamic coefficients. Though the iteration procedure is not clear still it gives a basic picture .It also gives the formulation of how to find frequency of vibration.

S Sarkar et al. [4] described the formulation of a rotor having misalignment in the bearing. It gave an idea of modeling the coupling misalignment in the finite element of the rotor .The displacement dependent stiffness for short bearings from static load deflection data was derived. An iteration method for static deflection of statically indeterminate finite rotor models was explained very clearly. After establishing the static equilibrium equations a simplified dynamic analysis was done with a numerical example.

M J Goodwin et al.[5] presented the experimental determination of the stiffness and damping coefficient and its validation with the numerical methods like Finite difference method. The formulation of numerical method for determination of stiffness and damping coefficients was given using perturbation technique. A test rig was established for determination of stiffness and damping coefficients. The perturbation was given by electromagnetic shaker. The results were analyzed and discussed very clearly. It also explained the governing equation of motion to find the frequency.

T H Machado and K L Calvacla [6] presented the solution of Reynolds equation for pressure field using finite difference method, but the procedure not clear. It gave pressure distribution for different types of bearings such as cylindrical bearing, elliptical bearing 3 lobed bearing etc. It gave the locus of shaft centre in the bearing but the procedure was unclear. However it gave a basic picture of finding the dynamic coefficients in different types of bearings. It also presented the solution for Reynolds equation including the squeeze effect.

J W Lund and K K Thomson [7] described how to calculate the stiffness an damping coefficient of oil lubricated using numerical method. It took the film rupture into account but the introduction of rupture was not clear. It gave a brief idea of how integration of pressure gives rise to load carrying capacity, four stiffness and four damping coefficients. Data were presented in tabular form for two axial groove bearings, elliptical bearings,3 lobed bearing and offset cylindrical bearings.

D Sfyris and A. Chasalevris [8] presented a picture of solution of Reynolds equation by using exact analytic solution. It used different mathematical tools and techniques like Bessel function, sturm-Liouville problem for eigen values power series method. It also used the 3-D CFD method for solving numerical validation. The detailed procedure was not understood clearly but one of the analytical solutions was realized to be possible in that context. It also validated the results with finite difference method.

O Ebrat et al. [9] discussed the solution of Reynolds equation including journal's elastic deformation. It used the finite difference method for solving the equation .The squeeze effect was considered and a superposition of wedging action and squeeze effect was taken. It considered the bearings with grooves for analysis .The dynamic coefficients were calculated using perturbation technique and using successive over-relaxation algorithm. To improve the computational accuracy, the concept of ‘influence zone’ and diagonalization procedure of stiffness and damping coefficients were included.

W B Rowe and F S Chang [10] used finite difference method for calculating pressure field from Reynolds equation based on which non-dimensional linearized dynamic coefficients were evaluated. Two techniques, Finite difference and perturbation technique were used for prediction of dynamic coefficients. Both analysis considered classical Reynolds equation with source of flow, film rupture and isoviscous fluid. This paper gave a clear picture of dynamic coefficients and detailed flowchart of the same, but the determination of static equilibrium position was not clearly mentioned.

L N Tao [11] used the strum-Liouville Theory and Heun’s equation to find the pressure distribution .The analytical method was used to solve the classical Reynolds equation without considering the squeeze effect. After the pressure field calculation the load carrying capacity and the coefficient of friction were calculated .However in this paper the boundary conditions involved in the solution of Reynolds equation that having very high importance was clearly discussed. The convergence of Heun’s function was also studied, as well as the eigen values and eigen functions were established.

Marco T .C Faria [12] presented the application of finite element method in the development of computer procedures for hydrodynamic elliptical journal bearing was studied. The finite element formulation for the classical Reynolds equation was made and the pressure field was calculated. The methods used here for the calculation of load capacity and the stability formulation was a bit unique. The procedure for finding the static equilibrium position was not clearly stated. How discretization affects the solution of Reynolds equation was also analyzed which was called mesh sensitivity analysis. The results obtained for finite finite element method was validated with other researchers. In the last step performance analysis of elliptical journal bearing was made.

R Tiwari [13] described classification of different types of fluid film bearings such as hydrostatic, squeeze film bearing etc were made very clearly. The finite difference solution of Reynolds equation was done in the literature for only wedging source term. It did not consider the squeeze effect. The procedure to find the static equilibrium position was not clearly mentioned..The empirical relation was made for calculation of Sommerfeld number and dynamic coefficients. The numerical method for finding the stiffness and damping coefficients were discussed but it was still unclear to develop a computer a computer program using it.

E Kramer [14] presented the derivation of classical Reynolds equation in a very clear and distinct manner. The sort bearing approximation of solving Reynolds equation was very clearly studied .The formulation and calculation of damping coefficients for short cylindrical bearings was clearly explained. The procedure to calculate the static equilibrium position was clearly mentioned.The Somerfield's solutions and boundary conditions was investigated. The empirical relation between the attitude angle and the eccentricity was studied. The static equilibrium curves for different types of bearings were studied.

V B Bhandari [15] presented how the journal moves in a hydrodynamic bearing in a particular pattern .The Raimondi Boyd solution was studied and attempt was made to compare to the original three papers. There was an easy and comprehensible derivation of the Reynolds equation in the literature. The iteration process in the Raimondi Boyd was not clearly understood .Different types of bearings and their working mechanisms were studied.

Richard G Budynas; J Keith Nisbett [16] showed the pattern of journal movement as a function of speed, viscosity eccentricity ratio etc. A comparative study was done between the texts learnt from Bhandari and Sighley. The Raimondi Boyd solution was again studied. There was a clear picture of different types of bearings, their functionalities and working. There was a scope of study the static equilibrium position of the journal in the bearing. No empirical relations were there. The short bearing and long bearing approximations were discussed which are called Sommerfeld's solutions. The working of journal in loaded and unloaded condition was clearly investigated.

Chapra [17] discussed the finite difference techniques for solving different types of partial differential equations .Other numerical tools like finite element method, finite volume method and their applicability were studied. The Gauss siedel over relaxation method was studied which is used in the present work.

Prasanta Sahoo [18] presented the derivation of Navier stokes equation and how it can be reduced to Reynolds equation using certain assumptions. The boundary conditions viz. Full Sommerfeld, Half Sommerfeld boundary conditions were studied. Different types of bearings such as Hydrostatic bearing, squeeze film bearing, elastohydrodynamic bearing were studied. The finite difference method to solution of Reynolds equation was investigated. The derivation of film thickness was studied.

Ping Huang [19] discussed different types bearings .The formulation of Reynolds equation was studied. How the Reynolds equation governs the fluid flow in various bearings was analyzed. The algorithm to solve modified Reynolds equation for different types of bearings with Fotran code was studied. The procedures and algorithms were very unclear.

H. Hirani [20] presented the derivation of Navier stokes equation and how it can be reduced to Reynolds equation using certain assumptions. The boundary conditions viz. Full Sommerfeld, Half Sommerfeld boundary conditions were studied. Different types of bearings such as Hydrostatic bearing squeeze film bearing, elastohydrodynamic bearing were studied .The finite difference method to solution of Reynolds equation was investigated. The derivation of film thickness was studied.

W. Stachowiak and Andrew W. Batchelor [21] presented the formulation and derivation of Reynolds equation. The misalignment parameter was considered in the expression of film thickness in Reynolds equation. The solution of classical Reynolds equation was done using finite difference method but another parameter called Vogelpohl parameter was introduced..The pressure field was calculated using Vogelpohl parameter for partial and grooved bearings. The dynamic coefficients were calculated but the source of exciter mass was unclear in the literature.

From different literatures stated above the basic idea about the solution of the Reynolds equation was gained. Different numerical methods like FDM, FEM, and analytical methods were understood. It was evident from the literature that for solving any rotor dynamics problem that includes the hydrodynamic bearing the stiffness and damping coefficients are necessary and for which solution of Reynolds equation and finding the equilibrium position is an inevitable task.

Chapter 3

METHODOLOGY

This chapter named as methodology gives an idea of the mathematical formulation and steps to solve the Reynolds equation for pressure distribution and its post processing for determining the equilibrium positions.

3.1. Hydrodynamic Bearing Theory

There are three components in a journal bearing assembly viz. bearing shell, journal and lubricant The journal rotates with some angular velocity and is statically or dynamically loaded in the radial direction. The bearing shell is rigidly supported. Under loaded condition the journal takes eccentric positions which gives rise to a convergent-divergent path in which when the lubricant flows generates a pressure.

The theory of fluid film lubrication is based on a differential equation derived by Osborne Reynolds. This equation is based on the following assumptions.

- The lubricant obeys Newton's law of viscosity.
- The lubricant is incompressible.
- The inertia forces in the oil film are negligible.
- The shaft and bearing are rigid.
- There is a constant supply of lubricant.

The simplified Reynolds equation is given by

$$\frac{\partial}{\partial x}\left(h^{3}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^{3}\frac{\partial p}{\partial y}\right)= 6\mu U\frac{\partial h}{\partial x} \qquad (3.1)$$

Exact analytical solution is not possible for equation for bearings with finite length. Theoretically exact solutions are possible if the bearing is assumed to be either very long or very short. These two solutions are popularly known as Somerfield's solution.

In this chapter a popular numerical technique 'Finite difference method ' is introduced and its application to analysis of hydrodynamic lubrication is demonstrated.

3.2. Non-dimensionalization of Reynolds equation

This process gives the generality of a numerical solution as well as the contribution of each term in the Reynolds equation.

$$h^{*} = \frac{h}{c} = 1 + \cos(\theta - \beta), \quad x^{*} = \frac{x}{R}, y^{*} == \frac{y}{L}, \; p^{*} == \frac{pc^{2}}{6U\mu R}, \; \varepsilon = \frac{e}{c},$$

The Reynolds equation in its non-dimensional form

$$\frac{\partial}{\partial x}(h^{*3}\frac{\partial p}{\partial x})+\frac{\partial}{\partial x}(h^{*3}\frac{\partial p}{\partial x})=6\mu U\frac{\partial h^{*}}{\partial x^{*}} \quad (3.2)$$

3.3 The vogelpohl parameter

The vogelpohl parameter is introduced for the accuracy of numerical solutions of the Reynolds equation .The Vogelpohl parameter M_v is defined as follows.

$$M_v = p^{*}h^{1.5} \quad (3.3)$$

Substitution into the non-dimensional form of the Reynolds equation yields the Vogelpohl equation

$$\frac{\partial^2 M_v}{\partial x^{*2}} + (\frac{R}{L})^2\frac{\partial^2 M_v}{\partial x^{*2}} = FM_v+G \quad (3.4)$$

Where parameters ‘F’ and ‘G’ for journal bearings are as follows

$$F= \frac{0.75[(\frac{\partial h^{*}}{\partial x^{*}})^2+\left(\frac{R}{L}\right)^2(\frac{\partial h^{*}}{\partial x^{*}})^2]}{h^{*2}} + \frac{1.5[\frac{\partial^2 h^{*}}{\partial x^{*2}}+(\frac{R}{L})^2\frac{\partial^2 h^{*}}{\partial y^{*2}}]}{h^{*}} \quad (3.5)$$

$$G=\frac{\frac{\partial h^{*}}{\partial x^{*}}}{h^{*1.5}} \quad (3.6)$$

The vogelpohl parameter makes the computing easy by simplifying the differential operators of Reynolds equation.

Numerical solution of the Reynolds equation are obtained in terms of M_v and values of p^{*} found from the definition $M_v/h^{*1.5}$.

3.4. Finite difference equivalent of Reynolds equation

The finite difference is based on the approximation of the differential quantity by the difference of functional values at two different nodes.

$$\left(\frac{\partial M_V}{\partial x^*}\right)_i = \frac{M_{v,i+1} - M_{v,i-1}}{2\delta x^*} \tag{3.7}$$

where the subscripts i-1 and i+1 denote position one step back and one step forward of the central position i and δx^* is the step length between nodes.

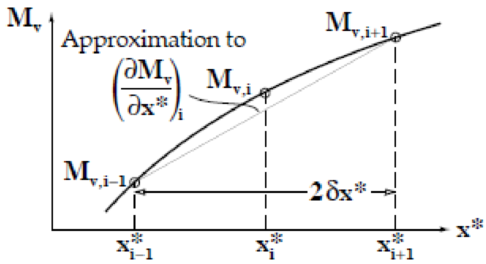


Fig 3.1 Nodal determination of Vogelpohl parameter

The finite difference is based on the approximation of the differential quantity by the difference of functional values at two different nodes.

$$\left(\frac{\partial M_V}{\partial x^*}\right)_i = \frac{M_{v,i+1} - M_{v,i-1}}{2\delta x^*} \tag{3.8}$$

where the subscripts i-1 and i+1 denote position one step back and one step forward of the central position i and δx^* is the step length between nodes.

$$\left(\frac{\partial^2 M_v}{\partial x^{*2}}\right)_i = \frac{\left(\frac{\partial M_v}{\partial x^*}\right)_{i+0.5} - \left(\frac{\partial M_v}{\partial x^*}\right)_{i-0.5}}{\delta x^*} \quad (3.9)$$

Where

$$\left(\frac{\partial M_V}{\partial x^*}\right)_{i+0.5} = \frac{M_{v,i+1} - M_{v,i}}{\delta x^*}$$

$$\left(\frac{\partial M_V}{\partial x^*}\right)_{i+0.5} = \frac{M_{v,i+1} - M_{v,i-1}}{\delta x^*}$$

Substituting into (5.7) gives

$$\left(\frac{\partial^2 M_v}{\partial x^{*2}}\right)_i = \frac{M_{v,i+1} + M_{v,i-1} - 2M_{v,i}}{\delta x^{*2}}$$

The terms 'F' and 'G' can be put in the finite difference equation to form the modified equation of the Reynolds equation and after rearranging the expression for '$M_{v,i,j}$' can be obtained as

$$M_{v,i,j} = \frac{C_1(M_{v,i+1,j} + M_{v,i-1,j}) + \left(\frac{R}{L}\right)^2 C_2(M_{v,i,j+1} + M_{v,i-1,j}) - Gi,j}{2C_1 + 2C_2 + Gi,j} \quad (3.10)$$

Where $C_1 = \frac{1}{\delta x^{*2}}$,

$C_2 = \frac{1}{\delta y^{*2}}$

3.5. Boundary conditions ,initial conditions and the solution domain

3.5.1. Boundary conditions

Pressure or the Vogelpohl parameter at the edges or the bearings is i.e. $P^*_{y=\pm 0.5}=0$ or M_v at y=±0.5

3.5.2.Initial conditions

Pressure or the vogelpohl parameter at all nodes were initially assumed to be zero.

i.e. $P^*=0$ or $M_v=0$ for all nodes

3.4.3. Solution domain

The bearing surface is discritized in two axis . One axis varies from 0 to 2πR on angular cordinate. Another axis varies from $-\frac{L}{2}$ to $\frac{L}{2}$. After nondimensionalization one axis varies from 0 to 2π and other axis varies from -0.5 to 0.5.

3.6. Solution Procedure for Reynolds equation

Analysing the equation it is evident that pressure at any gridpoint (i,j) can be expressed in terms of pressure of four adjacent nodes.The pressure(or the vogelpohl parameter) was assumed to be zero in first iteration. In order to obtain the pressure values at each node equation (3.7) .As the lubricant goes on flowing,the pressure gets developed at each node.Each node has a new value of pressure with new iteration.For covergence ,Gauss Siedel over relaxation criteria is used. For more clarification a simple flow chart is given below.

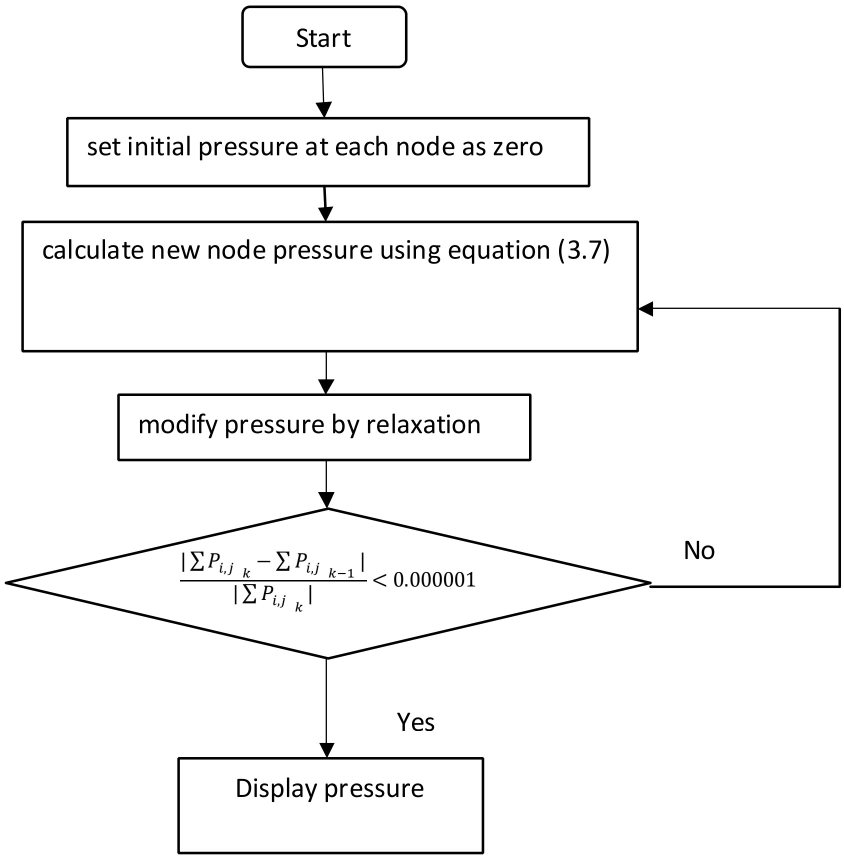


Fig 3.2 Flowchart to solve the Reynolds equation

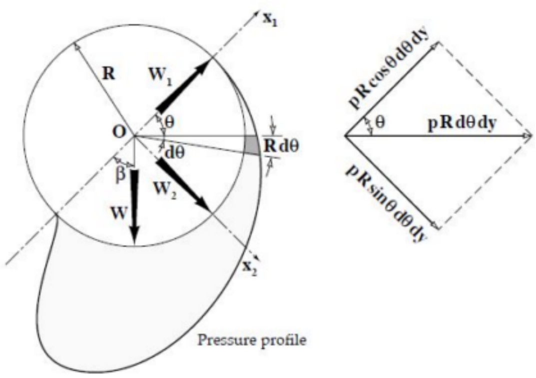


Fig 3.3 Forces of Lubricant

The total load which the bearing will support can be obtained by integrating the pressure around the bearing. Load is generally calculated from two compnents,one acting along the line joining the centres of journal and bearing and the second component acting perpendicular to the first.This method gives a scope of finding the angle between the centres .

The shaft doesnot always move co-directionlly with the load line,instead makes an angle to the load lin e.The angle is as the '*attitude angl(β)*'. It gives the position of minimum film thickness .

To derive expressions for the load components W_1 and W_2 a small element of area $Rd\theta dy$ has been taken where y axis is perpendicular to plane of paper.

The force exerted by the the pressure on the small area is $Rd\theta dy$ can be resolved into two components.

.$pR\cos\theta d\theta dy$ acting along the line joining shaft centre a bearing centre

.$pR\sin\theta d\theta dy$ acting perpendicular the line joining shaft centre a bearing centre

Axial force $W_1=\int_0^{2\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} pR\cos\theta d\theta dy$

Transverse force $W_2=\int_0^{2\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} pR\sin\theta d\theta dy$

The equilibrium position can be obtained by finding the attitude angle .In other words for a particular speed and external load journal attains an equilibrium position which is given by a set of values of eccentricity ratio and attitude angle.

The attitude angle is given by

$\tan\beta = -\frac{W2}{W1}$ where β is the attitude angle

3.7. Determination of attitude angle by iteration method

The attitude angle was first set to zero value in the film thickness equation.Then for each value of attitude angle the pressure is determined.From the pressure distribution the dimensionless load is calculated.When the transverse force is very less as compared to the axial force that angle is set as the attitude angle.Otherwise the process is continued upto convergence.

Chapter-4

RESULTS AND DISCUSSIONS

This chapter is the most important part of the thesis named as '**Results and Discussions'** where the results are discussed.

4.1 Pressure distribution

The pressure distribution of the fluid film inside the bearing is discussed in this section Analysis was made to cylindrical bearing of finite width. The variation of dimensionless pressure is discussed .The variation of pressure along the circumferential direction as well as the axial length has been shown. i.e. p* = f(x,y). For further clarification of the values at each point in the circumferential direction, the pressure distribution at the central line of the distribution of pressure has also been shown .The same work has been validated with Zhou et al [1].

Table4.1. Calculating parameters of journal bearing

Symbol	c,mm	L,m	R,m	F_W,N	N,rev/min	μ Nsm^{-2}	m	n
Value	0.6	0.12	0.1	1962	3000	0.04	60	21

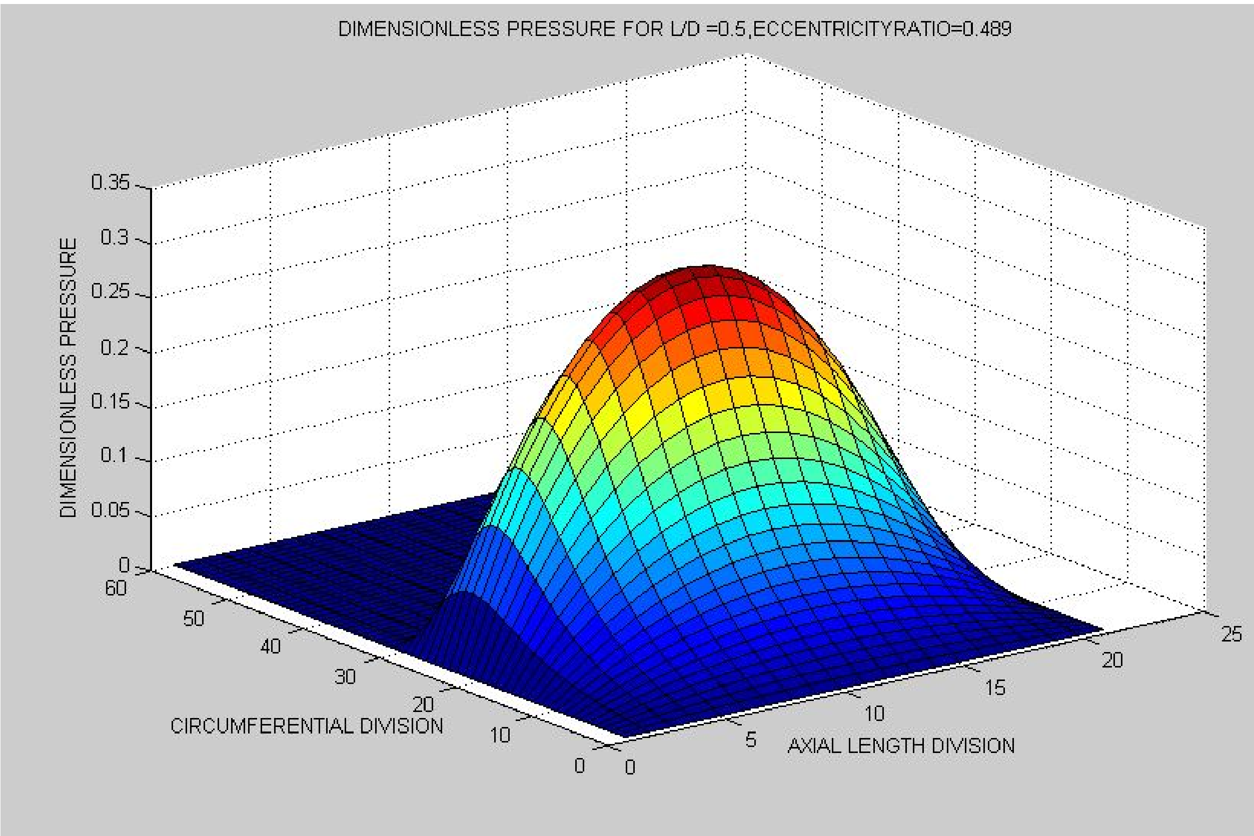


Fig 4.1. Pressure distribution for finite difference method for L/D ratio 0.5 and eccentricity0.489 [18]

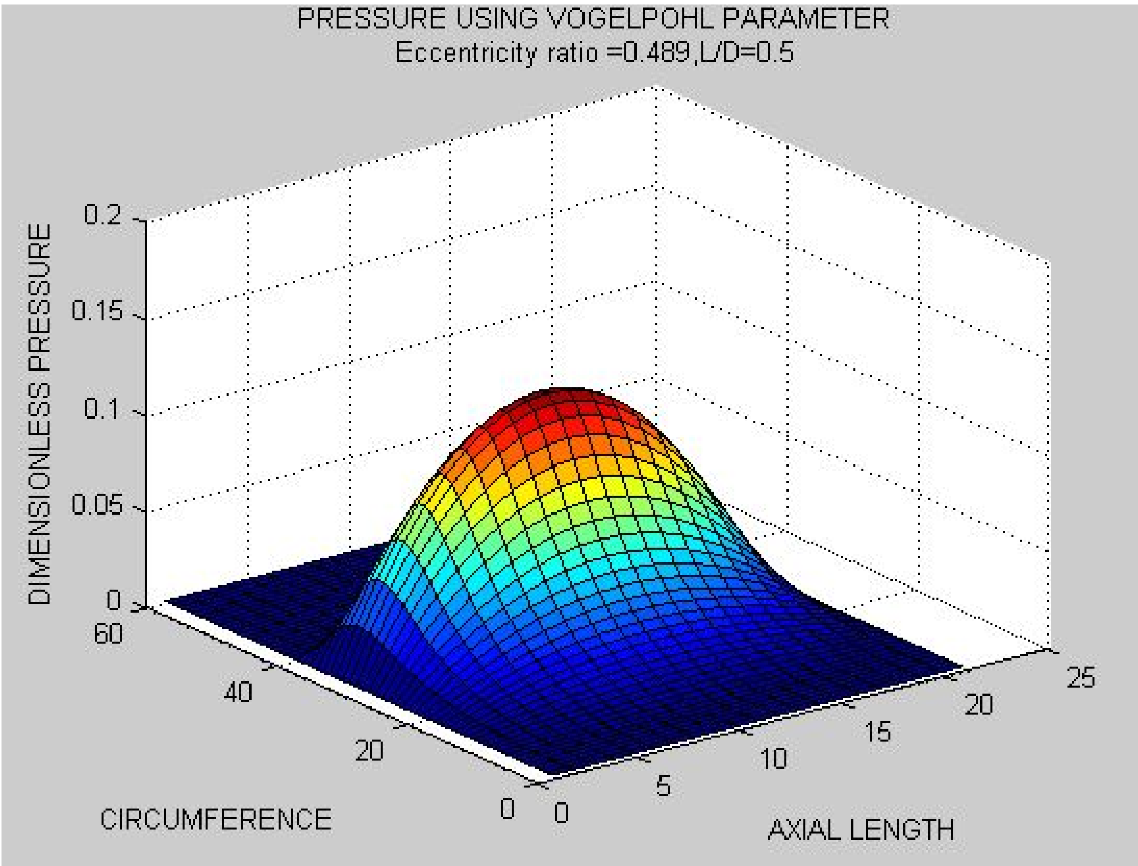


Fig 4.1. Pressure distribution for finite difference method(using Vogel parameter) for L/D ratio 0.5 and eccentricity 0.489 [20]

For validation purpose two sets of and eccentricities and L/D ratios has been taken that is to examine that the iteration procedure followed in the work is justified to certain limit.

It can be noted that the dimensionless pressure only depends on eccentricity ratio and the L/D ratio. The surface plot and the plot for central line pressure has been shown in Fig 4.1 and Fig 4.2

The pressure distribution in the entire solution domain shows that the positive pressure is there in the convergent wedge .The convergent wedge appears to extend up to a the mid portion of the bearing sequentially,after which the lubricant enters the divergent wedge .The divergent wedge generally starts from the mid portion of the bearing and extends up to the end . The values changes from positive to negative due to expansion effect. Axial end of the fluid film surface the pressure is equal to environmental pressure (atmospheric pressure).Hence the relative pressure is zero.

Negative pressure within the film is considered equal to fluid vapor pressure (atmospheric pressure).Hence relative pressure is also zero here.

In fact the actual lubricant cannot sustain tensile stress and liquid film will fracture in the divergent wedge. That is why Half Summerfield boundary conditions are generally used.

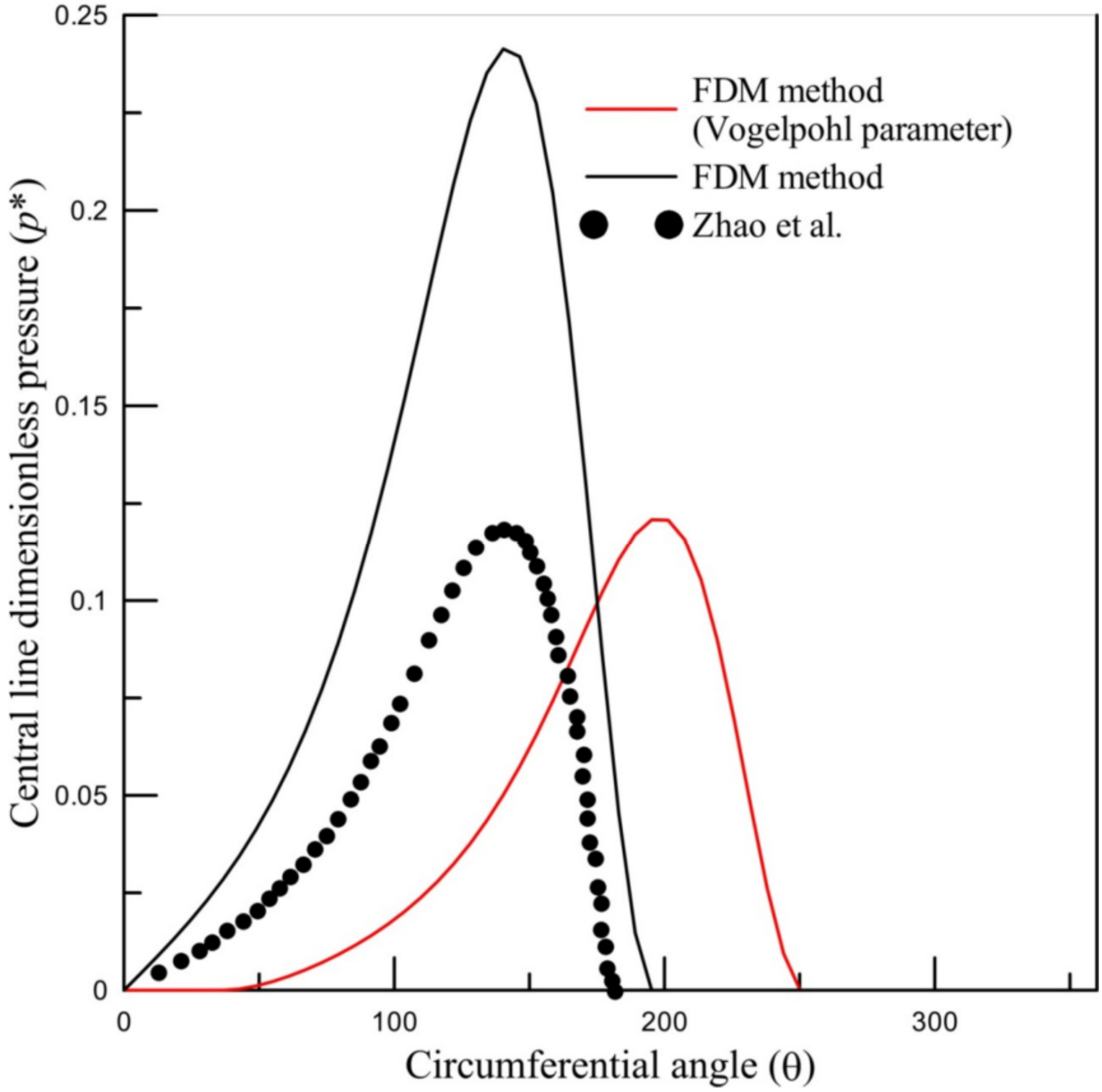


Fig 4.3 Validation of pressure distribution using Finite difference method

The inception of the negative pressure zone is found to be different in Fig 4.3 for different methods though the input parameters were set to be equal. This is because there was no scope to use the boundary condition for the mid circumferential plane of the bearing. The initial pressure was assumed to be zero and the pressure was allowed to develop. So there is no provision of the start of the negative pressure zone at the same point for all three curves.

Compared to the Finite difference method formulated in [18] ,the black curve in Fig 4.3 ,the introduction of Vogelpohl parameter for solving the classical Reynolds equation goes in good accordance with Zhou et al [1]. Therefore, the latter has been considered further analysis and finding the static equilibrium position.

The pressure distribution for the bearings for a square bearing for different values of eccentricity ratios have been shown in the following figures.

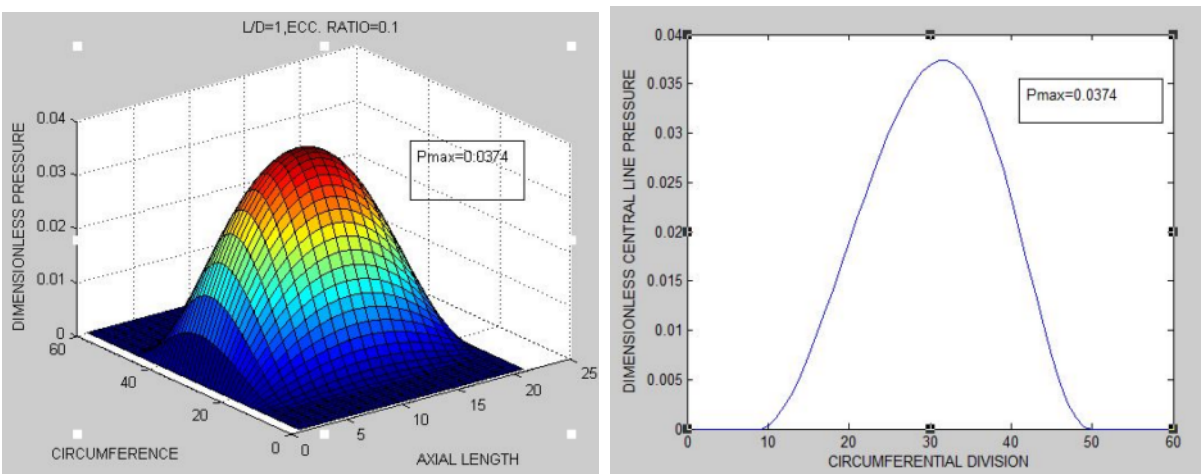


Fig 4.4 Pressure distribution for L/D=1, Eccentricity ratio=0.1

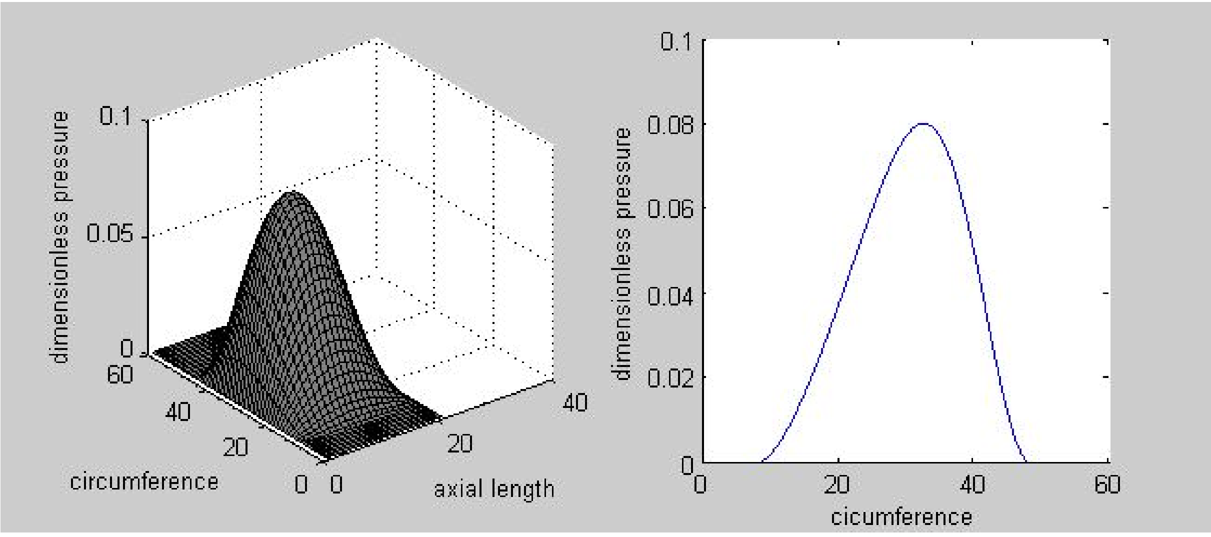


Figure 4.5 pressure distribution for L/D=1 and eccentricity ratio=0.2

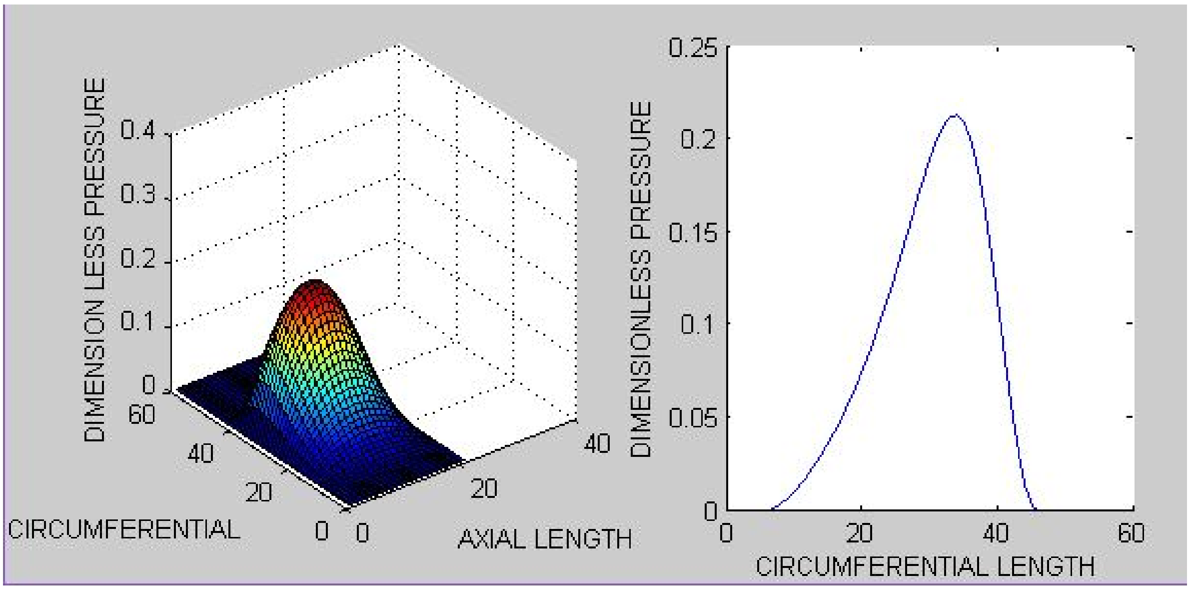


Figure 4.6 pressure distribution for L/D=1 and eccentricity ratio=0.4

From Figure 4.4, Fig 4.5 and Fig 4.6, it is clearly evident that the pressure goes on increasing from 0.0374 to 0.2 as we increase the eccentricity values. With increase in pressure values the load carrying capacity also increases.

4.2 Static Equilibrium position

The static equilibrium position in a bearing is characterized by its eccentricity ratio and the attitude angle has been obtained from an iterative treatment from W. Stachowiak and Andrew W. Batchelor [21] which has been discussed in chapter 3. The same work has been validated with Zhou et al. [1] which uses two fold secant method.The input parameters used for determining the static equilibrium position has been presented in table 4.1.The load carrying capacity and the influence of the working variables has been discussed.

4.3 Effect of Geometric parameters on the static equilibrium position

To investigate the influence of geometrical parameters and working variables on the equilibrium positions the bearing length 'L' ,clearance' c' and rotational speed ' N' were selected as independent variables.

The parameters which are used to study the equilibrium position are given in table except bearing length because the bearing length is varied to study the equilibrium positions.

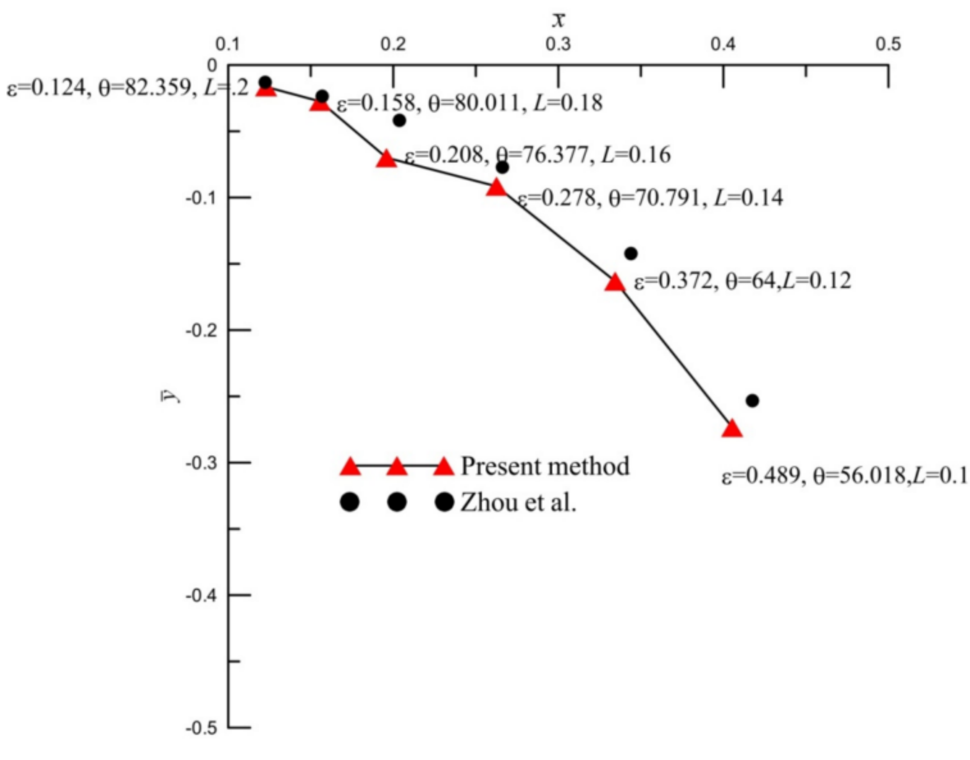


Fig 4.7 Effect of Bearing length (L) on the equilibrium position

It is clearly seen that the equilibrium position slowly approaches the centre of the bearing. This can be observed from the variation of attitude angle and eccentricity. The eccentricity ratio 0.489 to 0.124, where as the attitude angle increases from 56.018 to 82.359. This result is consistent with Zhou et al. [1].

The corresponding fitting curve of the eccentricity and attitude angle is plotted in Fig 4.8.The straight line is fitted by least square method. The relationship between the attitude angle and eccentricity was found to be linear with respect to bearing length.

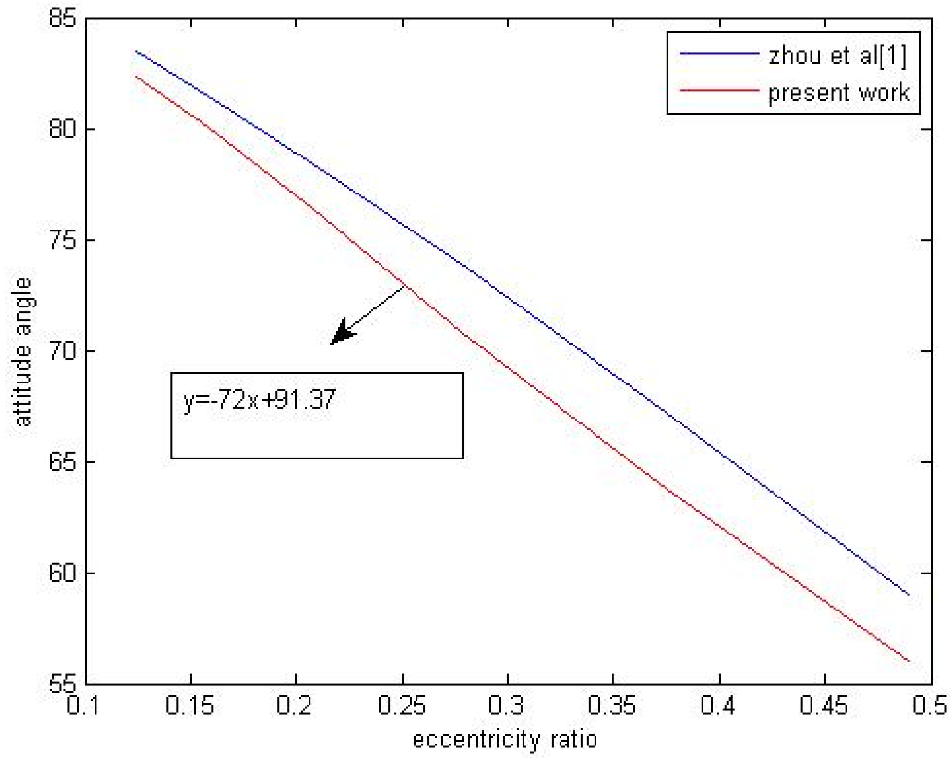
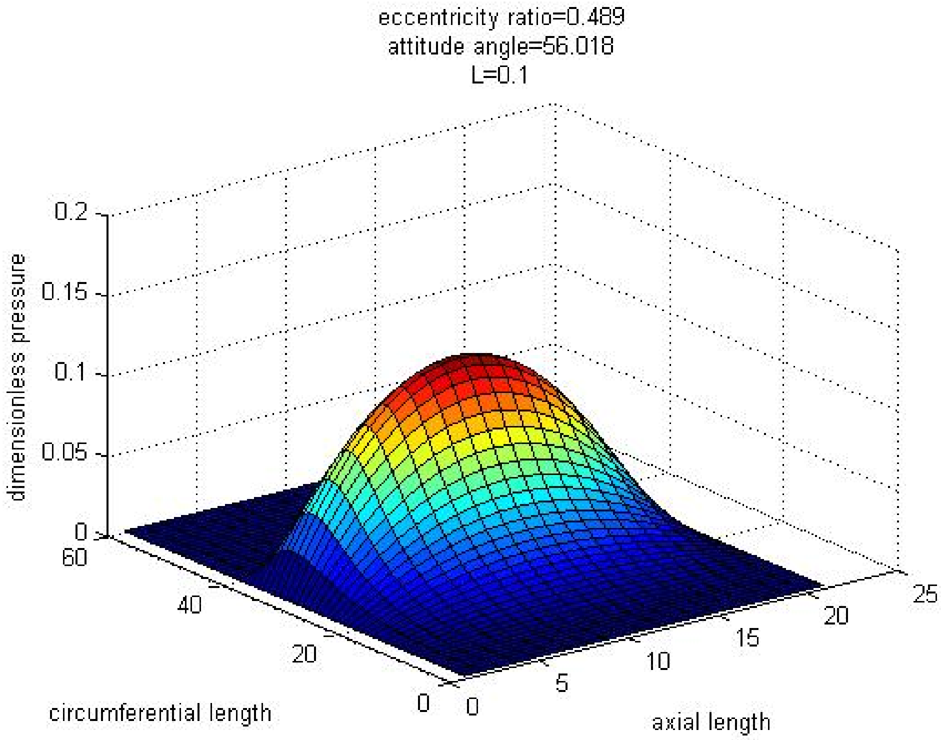
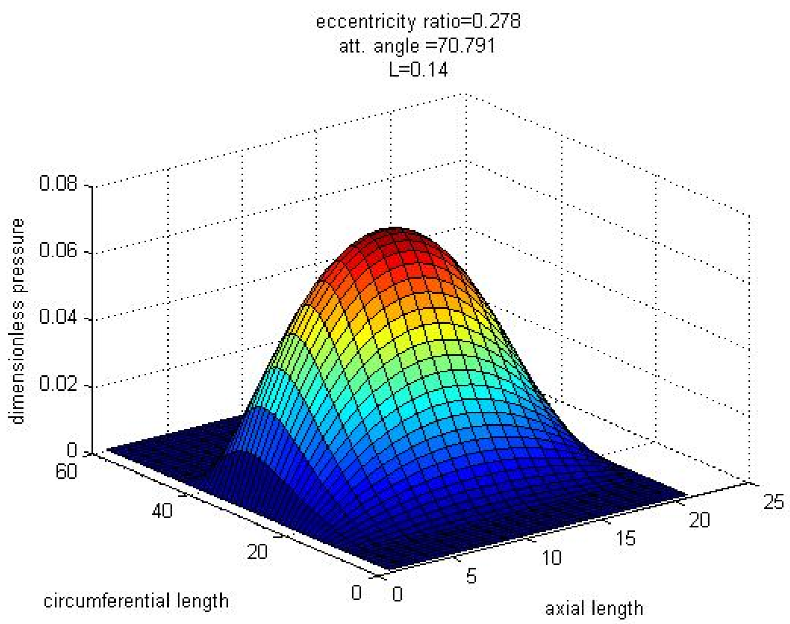


Fig 4.8 Fitting curve for eccentricity and attitude angle

Fig 4.9 and Table 4.2 represents the pressure distribution curves and maximum values of the dimensionless pressure for corresponding equilibrium positions shown in Fig 4.9 shows that peak pressure decreases as the bearing length goes on increasing. The maximum dimensionless pressure of 0.118 is obtained when L=0.1. The smaller the bearing length and larger the eccentricity ratio the flow clearance is decreased which says that the bearing with smaller length can reduce pressure leakage and sustain pressure better than the bearing with larger length. When bearing length increases from 0.1 to 0.2 i.e. .for smaller bearing length equilibrium position of the maximum pressure gets nearer to the minimum film thickness.





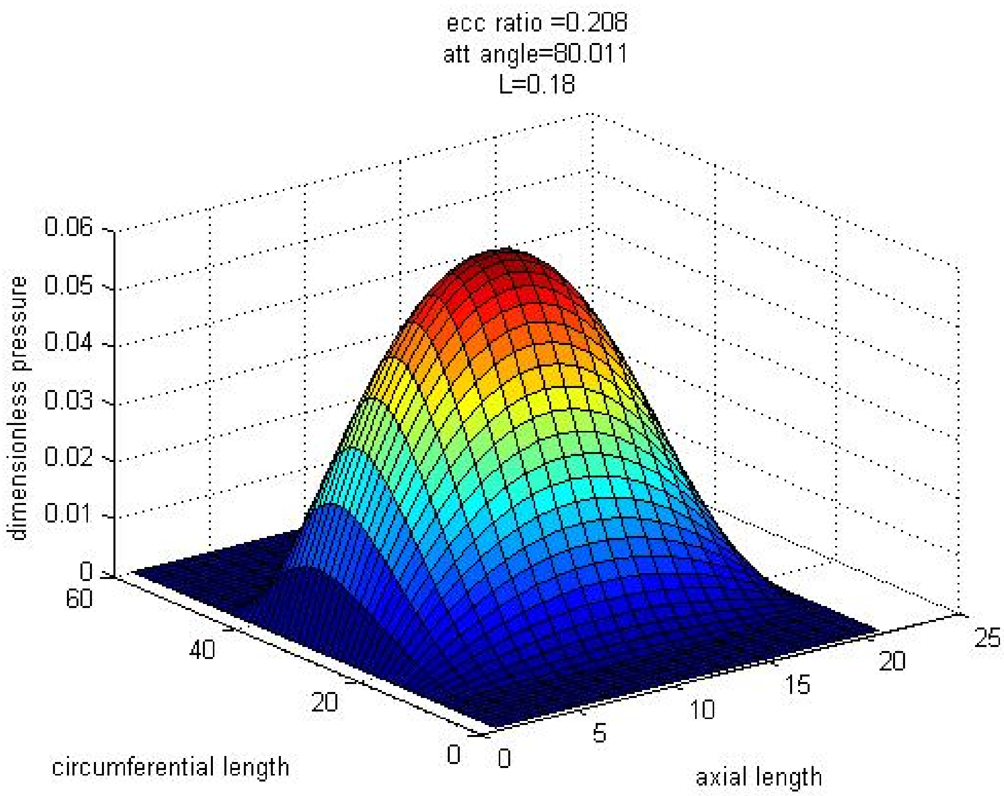


Fig 4.9 Pressure distribution with different L

Table 4.2 . Variation of maximum pressure with bearing length

Bearing length	Maximum dimensionless pressure
0.10	0.12
0.12	0.088
0.14	0.0556
0.16	0.05995
0.180	0.052118
0.2	0.046

Chapter 5

CONCLUSION
AND
FUTURE SCOPE OF WORK.

5.1. Conclusion

In the present work The Reynolds equation for stable motion was solved numerically using finite difference method. The results were validated with Zhou et al. [1]. Using the pressure distribution values the static equilibrium position of the journal was calculated and the results were found to be in consistent with the values of Zhou et al. [1] The effects of geometric parameters(length) and working variables on equilibrium positions ,pressure distribution and load carrying capacity was investigated.

The main conclusions are as follows

- The pressure was found to be positive in the convergent region where as found negative in the divergent zone which was made zero. The pressure distribution was found to be symmetric about the central -line of the bearing.
- The trajectory of equilibrium positions was found to be parabolic and the relation between the attitude angle and the eccentricity was found to be linear when fitted using least square method.

5.2. Scope for future work

- How the equilibrium position is affected by other parameters like clearance and rotational speed can be studied.
- Using the static equilibrium positions the stiffness and damping coefficients can be found out and the dynamic analysis of the rotor in the bearing can be done.

Chapter 6

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APPENDIX

Program1: Solution of Reynolds equation using finite differnce method

```
clc
clear all
%----------------------
%preparameter assignment
B=0.12;%bearing length
R=0.1;%radius of journal
C0=0.6e-3;%radial clearnance
AN=3000;%rotational speed in
EDA=0.04;%viscocity
EPSON=0.489;
N=60;%Number of nodes in radial direction
M=21;%number of nodes in axial direction
dx=(2*pi)/(N-1);
dy=1/(M-1);
OMEGA=AN*2*pi/60;
U=OMEGA*R;
ALFA=((R/B)*(dx/dy))^2;
sum(1)=0;
sumij=0;
iter=1500;
%----------------
%calculation of film thickness
for i=1:N
   SETA=(i-1)*dx;
   for j=1:M
      h(i,j)=1+EPSON*cos(SETA);
   end
end
%--------------------------
%assignment of initial and boundary conditions
for i=1:N
   for j=2:M-1
      p(i,j)=0.5;
   end
end
for j=1:M
   p(1,j)=0;
   p(N,j)=0;
end
for i=1:N
   p(i,1)=0;
   p(1,M)=0;
end
%calculation of pressure
for ik=1:15000
c1=0;
aload=0;
for i=2:N-1
   i1=i-1;
```

```
    i2=i+1;
    for j=2:M-1
        pd=p(i,j);
        j1=j-1;
        j2=j+1;
        A1=(0.5*(h(i2,j)+h(i,j)))^3;
        A2=(0.5*(h(i,j)+h(i1,j)))^3;
        A3=ALFA*(0.5*(h(i,j2)+h(i,j)))^3;
        A4=ALFA*(0.5*(h(i,j)+h(i,j1)))^3;
        p(i,j)=(-dx*(h(i2,j)-
h(i1,j))+A1*p(i2,j)+A2*p(i1,j)+A3*p(i,j2)+A4*p(i,j1))/(A1+A2+A3+A4);
      p(i,j)=0.7*pd+0.3*p(i,j);
      if p(i,j)<0
          p(i,j)=0;
      end
      c1=c1+abs(p(i,j)-pd);
      aload=aload+p(i,j);
    end
end
c1=c1/aload;
if c1<1e-7
    break
end
end
y = ik
surf(p)
```