Ref. No : *Ex/EE/T/422A/2019*

Bachelor of Electrical Engineering Examination, 2019
4th Year, 2nd Semester

Advanced Control Theory

Time: Three Hours; Full Marks: 100

Answer both parts on the same answer script

Part-I

Answer any *three* questions from this part (all questions carry equal marks)

Two marks for neat and well-organized answerscript

1. a) Discuss the common sources and types of nonlinearity in plants. **6+4+2+4**
   b) What is static non-linearity? Explain clearly.
   c) Give two examples of static nonlinearity.
   d) (i) What is hysteresis type of nonlinear characteristics? (ii) Why is it called a nonlinearity with memory?

2. a) Explain what is meant by "Equilibrium Point" of a nonlinear dynamic system. **2+(6+4+4)**
   b) A nonlinear system is expressed as follows:

$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_2 = 0.1x_1 - 2x_2 - x_1^2 - 0.1x_1^3$$

   (i). Determine the equilibrium points of the above system.
   (ii). Linearize the above system about its equilibrium point at $\mathbf{x}=0$.
   (iii). Comment about the asymptotic stability of the system at $\mathbf{x}=0$ by using Lyapunov's first theorem or any other suitable method.

3. a) State Lyapunov's 2nd theorem. Briefly describe how this theorem may be used to determine the stability of a nonlinear dynamic system. What are its limitations? **6+10**
   b) The dynamics of an unforced nonlinear system is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - x_1\left(x_1^2 + x_2^2\right) \\ -x_1 - x_2\left(x_1^2 + x_2^2\right) \end{bmatrix}$$

. Using the function $V = \frac{1}{2}\left(x_2^2 + x_1^2\right)$ as the Lyapunov function, investigate the stability of the system about its equilibrium point at the origin.

[Turn over

4. a) Enumerate the advantages and disadvantages of on-off control. **2+4+4+6**
   b) With schematic diagrams explain how an on-off type temperature control system functions. State the necessary controller characteristics.
   c) What is a phase plane plot? How does it help stability analysis of nonlinear systems? Explain the meaning and use of isoclines for this plot.
   d) With suitable phase plane diagrams discuss how the stability of standard second order systems with different pole locations may be analyzed by their phase portraits.

5. a) The output of an ideal (without hysteresis) bi-directional relay element is as follows: **6+10**

$$y(\omega t) = \begin{cases} +1; & \forall 0 < \omega t < \pi \\ -1; & \forall \pi < \omega t \le 2\pi \end{cases}.$$

   Find the describing function for the above relay element.
   b) A closed loop unity feedback control system has the relay element described in 5(a) above, along with a linear process with transfer function $G(s)\frac{1}{s(s+1)^2}$ in the forward path. Investigate the stability of the closed loop nonlinear control system by using the describing function obtained in 5(a) and comment about the presence of limit cycle, if any, on the basis of the above analysis.

Part II

Answer any *three* questions from this part (all questions carry equal marks)

Two marks for neat and well-organized answerscript

6. a) Explain the difference between the terms 'Structured uncertainty' and 'Unstructured uncertainty. **4+12**
   b) Check for the robust stability of the system whose characteristic polynomial is given by

$$p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 = 0,$$

   where

$$p_5 \in [1, 1],\ p_4 \in [35, 40],\ p_3 \in [61, 64],\ p_2 \in [35, 36],\ p_1 \in [11, 15]$$

   and $p_0 \in [52, 58]$.

7. a) A process plant given by $G_1(s) = \dfrac{2}{(s+1)(0.01s+1)}$ is modeled by using the transfer function $G_2(s) = \dfrac{2}{s+1}$. Compare (i) the open loop unit step responses, and (ii) the frequency responses of the plant and its model. **(2+4)+5 +5**

   b) Given a transfer function $G(s) = \dfrac{16}{(s+1)(s+2)^2(s+4)}$. Find $\|G\|_2$.

   c) For the system with transfer function $G(s) = \dfrac{0.2s+1}{s+1}$, find $\|G\|_\infty$.

8. a) What is an observer? What are its uses? Explain with the help of a block diagram. **6+10**

   b) Design a reduced order observer for observing the second state variable for the following continuous time system so that the observer pole is located at -10,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}u \; ; \quad y = [1 \;\; 0]\,\mathbf{x}\,.$$

9. a) What is optimal control? Explain the meaning of the term 'quadratic performance index'. **4+10+2**

   b) Explain what is meant by the following terms giving an example in each case:
   (i) The tracking control problem
   (ii) The regulator control problem
   (iii)The terminal control problem
   (iv) The minimum-time control problem
   (v) The minimum energy control problem

   c) State the expression for the Hamilton-Jacobi equation clearly mentioning all notations used.

10 A regulator contains a plant described by **6+6+4**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \; ; \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix}x$$

and has the performance index

$$J = \int_0^\infty \left[x^T \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x + u^2 \right] dt\,.$$

Determine
a) the Riccati matrix P
b) the optimal control law
c) the closed loop eigenvalues.