B.E. ELECTRICAL ENGINEERING THIRD YEAR SECOND SEMESTER EXAMINATION, 2019

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours (50 marks for each part)

Use a separate Answer-Script for each part

Question No.	PART- I	Marks
	Answer any *THREE* questions **Two marks reserved for neatness and well organized answers.**	
1. (a)	A box contains 15 carbon resistors and 5 metal oxide resistors. Two parts are drawn at random from the box. What is the probability that: (i) both are carbon resistors? (ii) both are metal oxide resistors? (iii) one is carbon resistor and the other is metal oxide resistor ?	4
(b)	Manufacturer X produces personal computers (PCs) at two different locations in the world. 15% of the PCs produced at location *A* are delivered defective to a retail outlet, while 5 % of the PCs produced at location *B* are delivered defective to the same retail store. If the manufacturing plant at *A* produces 1,000,000 PCs per year and the plant at B produces 150,000 PCs per year, find the probability of purchasing a defective PC.	5
(c)	Under what condition and in what manner can the cumulative distribution and the density functions of a sum of statistically independent random time variables (τ_1, τ_2, etc.) be obtained by performing time-domain operations involving the density functions $f(t_1)$, $f(t_2)$, etc. ? Give relevant derivations in support of your answer.	7

2. (a)	**Determine whether or not the following functions are valid cumulative distribution functions. Justify your answers.** (i) $G_X(x) = (1 - e^{-x/2})u(x)$ (ii) $G_X(x) = \frac{x}{a}[u(x-a) - u(x-2a)]$	4
(b)	**Does the sum of M number of *independent* exponential random variables with same distribution function, represent any other well-known variety of random variable? Explain with the help of relevant derivation.**	6
	OR	
	The lifetimes of six major components in a copier are independent exponential random variables with means of 8000, 10,000, 10,000, 20,000, 20,000, and 25,000 hours, respectively. **(i) What is the probability that the lifetimes of all the components exceed 5000 hours?** **(ii) What is the probability that at least one component lifetime exceeds 25,000 hours?**	6
(c)	**Define '*Characteristic Function*' of random variables.** **What characteristic features of a random variable can this function yield? Explain, proceeding mathematically from the basic definition. Illustrate the application for a uniformly distributed random variable.**	6
3. (a)	**The joint density function of random variables X and Y is** $f(x,y) = xe^{-x(1+y)} \quad x \geq 0 \text{ and } y \geq 0$ $= 0 \quad otherwise$ **Determine the marginal density functions.**	6

(b)	**Consider the bivariate probability density function** $$f_{X,Y}(x,y) = 2u(x)u(y)\exp[-(6y+\frac{x}{4})]$$ **Determine the expectations and the covariance of X and Y. Comment on the significance of the value of the covariance.**	6
(c)	**Two zero-mean discrete-time random processes $X[t]$ and $Y[t]$ are statistically independent. Let a new random process be $Z[t]= X[t] + Y[t]$. Let the autocorrelation functions for $X[t]$ and $Y[t]$ be** $$R_X[\tau] = \left(\frac{1}{2}\right)^{\|\tau\|} \text{ and } R_Y[\tau] = \left(\frac{1}{3}\right)^{\|\tau\|} \text{ respectively.}$$ **Find $R_Z[\tau]$.**	4
4. (a)	**An LTI system is excited by a WSS random process. Which property of the system relates the cross power spectral density of the output and the input to the power spectral density of the input? Give mathematical derivations starting from the convolution representation of the system.**	6
(b)	**A wide-sense stationary random process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-3\|\tau\|}$ is applied as input to a linear time-invariant (LTI) system with frequency response function** $$H(jf) = \frac{1}{9 + j2\pi f}$$ **Determine the expression for the autocorrelation function of the output random process.**	5
(c)	**The autocorrelation function for an ergodic random process is** $$R_{XX}(\tau) = 9 + \frac{4}{4+\tau^2}$$ **Find the mean value, the mean-square value, the variance and the power spectral density of the process $X(t)$. Give clear explanations.**	5

5.	**Write short notes on <u>any two</u> of the following.**	
(a)	**Johnson noise in resistors.**	**8+8**
(b)	**Probability generating function of random variables and its application.**	
(c)	**Poisson distribution.**	
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PART II

Whenever required, the following table for standard normal distribution may be used with interpolation/extrapolation if necessary. Wherever you use this table indicate clearly how you have used it.

z	0.95	0.98	1.00	1.02	1.90	1.94	1.96
$\phi(z)$	0.8289	0.8365	0.8413	0.8461	0.9713	0.9738	0.9750

1. a) X_i, $i=1,2,\ldots,n$ is a randomly chosen sample from a population with mean μ and variance σ^2. With necessary derivation and justification comment on the distribution of the sample mean when n is large. (6)

Or,

Show that the expected value of sample variance is equal to the population variance.

b) The weights of a population of workers have mean 165 kg and standard deviation 24 kg. If a sample of 64 workers is chosen, then calculate the probability that the average weight of these 64 workers lies between 162 kg and 168 kg. (4)

c) If $\overline{X}$ and S^2 are, respectively, the sample mean and sample variance for a randomly chosen sample of size n from a normal population with mean μ and variance σ^2, then show that $(n-1)S^2/\sigma^2$ has a chi-square distribution with $(n-1)$ degrees of freedom while $\sqrt{n}\left(\overline{X}-\mu\right)/S$ has a t- distribution with $(n-1)$ degrees of freedom. (8)

Or,

Write short notes on

(i) chi-square distribution (ii) t- distribution

2. a) From past experience it is known that the weight of fowls grown at a hatchery are normal with a standard deviation that remains fixed at 0.15 kg, but a mean that varies from season to season. With derivation of the necessary expressions determine how large a sample size is needed to be 95% confident that estimate of the present season's mean weight of a fowl is correct within ± 0.05 kg. (8)

[Turn over

b) Derive the expressions for the Least Square Estimators of the regression parameters of a simple linear regression model. Also show that these estimators are unbiased estimators. State clearly the assumptions made, if any. (8)

Or,

Write a brief note on statistical inferences about the regression parameters.

3.a) Define the following terms in relation with hypothesis testing: (10)
i) type-I error ii) type-II error iii) significance level
Develop a significance level α test for the following hypothesis.
H_0: $\mu = m$ against H_1: $\mu \neq m$
where μ indicates the mean of a normal population with known variance σ^2, and m is a specified constant.

b) Write brief notes on any one of the following: (6)

i) Probability of type II error in hypothesis test.
ii) One sided hypothesis test.
iii) Hypothesis test concerning the mean of a normal population when the variance is Unknown.