Ex/EE/Math/T/123/2019 (Old)

BACHELOR OF ENGINEERING IN (ELECTRICAL ENGINEERING) EXAMINATION, 2019

(1st Year, 2nd Semester)

MATHEMATICS - III F

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Symbols/Notations have their usual meanings :

Answer ***any five*** questions. 10×5=50

1. a) Show that the function $f(z) = u + iv$, where

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} (z \neq 0)$$

$$f(0) = 0$$

is continuous and Cauchy-Riemann equations are satisfied at the origin. But does not exist.

b) Define analytic function and singular point. 7+3

2. a) Find an analytic function whose real part is

$$u(x, y) = x^3 - 3xy^2.$$

[Turn over

b) show that an analytic function with constant modulus is constant. 5+5

3. a) Find rthe nature and location of singularity of $(z+1)\sin\frac{1}{z-2}$

b) State Residue theorem. Using residue formula evalute $\int_C \frac{(z-3)}{z^2+2z+5}dz$, where C is the circle

i) $|z|=1$, ii) $|z+1-i|=2$ 4+6

4. a) Expand $f(z)=\frac{1}{(z-1)(z-2)}$ in Laurent's series in the region

i) $1<|z|<2$,

ii) $0<|z-1|<1$.

b) Evaluate the integral,

$\int_C \frac{\text{Sin}^2 z}{\left(z-\pi/6\right)^3}dt$, where C : $|Z|=1$. 6+4

5. Solve the one dimensional heat equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ satisfying the conditions :

$u(0, t) = u(l, t) = 0$ and

$$u(x,0)=\begin{cases} x, & \text{if } 0\le x\le \frac{l}{2} \\ l-x, & \text{if } \frac{l}{2}\le x\le l \end{cases}$$ 10

6. Consider an infinite uniform plate bounded by the lines $x=0$, $x=a$ and $y=0$. The edge $y=0$ is maintained at temperature f(x) and the remaining edges at temperature 0^o. Find the temperature distribution u(x, y) from the equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0.$$ 10

7. Find the solution in series about for the differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$ 10

PART - II

Symbols/Notations have their usual meanings :

Answer ***any five*** questions.

8. a) Find Laplace transform of (i) (ii) cos at sinhat

b) If $L\left[2\sqrt{\frac{t}{\pi}}\right]=\frac{1}{p^{3/2}}$ then show that $L\left[\sqrt{\frac{t}{\pi t}}\right]=\frac{1}{p^{1/2}}$

6+4

9. a) Find the inverse Laplace transform of $\frac{3p+1}{(p-1)(p^2+1)}$

b) Use convolution theorem to find $L^{-1}\left[\frac{1}{(p^2+a^2)^2}\right]$ 5+5

10. a) Find the Fourier transform of $f(x)$ defined by

$$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0 & |x|>1\end{cases}$$

b) Find Fourier sine transform of $2e^{-5x}+5e^{-2x}$. 6+4

11. a) Find the inverse Fourier transform of $\frac{\xi}{1+\xi^2}$

b) Find Fourier cosine transform of te^{-at}. 6+4

12. Solve the following differential equation by using Laplace transform :

$$\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=0,\ \ y(0)=1,\ \ y'(0)=y''(0)=2.$$

10

13. Prove that

$$\int_{-1}^{1}P_m(x)P_n(x)dx=0,\ \text{if}\ \ m\neq n$$

$$=\frac{2}{2n+1}\quad \text{if}\ \ m=n$$ 5+5

14. Prove that

i) $\frac{d}{dx}\{x^nJ_n(x)\}=x^nJ_{n-1}(x)$

ii) $nP_n=xP_n'-P_{n-1}'$ 5+5