Bachelor of Engineering in Electrical Engineering Examination, 2019

(1st Year, 1st Semester, Old)

Mathematics - IIF

Time : Three hours   Full Marks : 100

(50 marks for each Part)

Use a separate Answer-Script for each Part

PART - I

(Answer question no. ***6*** and ***any three*** from the rest)

(Symbols/Notations have their usual meanings)

1. a) If $\varphi(x, y, z) = c$ represents a surface. Show that $\overline{\nabla}\varphi$ is a vector whose modulus is $\frac{\partial\varphi}{\partial\eta}$ (rate of change of φ along normal) and direction is along the normal to the surface 10

   b) If $\frac{d\overline{u}}{dt} = \overline{w} \times \overline{u}$, $\frac{d\overline{v}}{dt} = \overline{w} \times \overline{v}$,

   Show that $\frac{d}{dt}(\overline{u} \times \overline{v}) = \overline{w} \times (\overline{u} \times \overline{v})$ 6

2. a) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$. 8

[Turn over

b) Prove that 8

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \vec{A}\times(\vec{\nabla}\times\vec{B})$$

3. a) Verify Green's theorem in a plane to evaluate

$$\int_c [(3x - 8y^2)dx + (4y - 6xy)dy]$$

where c is the boundary of the region defined by $x = 0$, $y = 0$ and $x + y = 1$. 8

b) If u, v, w are orthogonal curvilinear co-ordinates show that $\frac{\partial \vec{r}}{\partial u}, \frac{\partial \vec{r}}{\partial v}, \frac{\partial \vec{r}}{\partial w}$ and $\vec{\nabla}u, \vec{\nabla}v, \vec{\nabla}w$ are reciprocal system of vectors. 8

4. State Gauss Divergence theorem.

Verify Gauss Divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\vec{j} + yz\vec{k}$ and S is the surface of the cube bounded by $x = 0, y = 0, z = 0$ and $x = a, y = a, z = a.$ 16

5. a) Express the vector $\vec{A} = 2y\hat{i} - z\vec{j} + 3x\vec{k}$, in spherical polar co-ordinates and determine $A_\gamma,\ A_\theta,\ A_\phi$. 10

b) Prove $\iint_s r^5 \vec{n} ds = \iiint_v 5r^3 \vec{r} dv$ 6

6. State Green's theorem. 2

b) Find the particular integral of

$$\frac{d^2y}{dx^2} + 4y = x^4 + \cos^2 x.$$ 5+5

13. Solve : $(x^2D^2 + 3xD + 5)y = x\cos(\log x) + 3,$

where $D \equiv \frac{d}{dx}$. 10

14. Explain the method of variation of parameters for solving a second order differential equation of the form

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

and hence solve

$$\frac{d^2y}{dx^2} + a^2y = \sec ax.$$ 10

b) Use cayley-Hamilton theorem to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$ 5+5

10. a) Solve, by matrix inversion, the equations

$$x + 2y + 3z = 14$$
$$2x - y + 5z = 15$$
$$2y + 4z - 3x = 13.$$

b) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, express $2A^5, -3A^4 - 4I$ as a linear polynomial in A. 5+5

11. a) Solve the differential equation

$$x \log x \frac{dy}{dx} + y = 2 \log x.$$

b) Find the integrating factor of the differential equation

$$(xy^3 + y)dx + (2x^2y^2 + 2x)dy = 0.$$ 5+5

12. a) Find the general solution of the differential equation

$$(D^4 - 1)y = e^x \cos x, \text{ where } D \equiv \frac{d}{dx}.$$

PART - II

Answer ***any five*** questions. 5×10

7. a) Show that

$$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc.$$

b) Factorise $\begin{vmatrix} \alpha & \beta & 1 \\ \alpha^2 & \beta^2 & 1 \\ \alpha^3 & \beta^3 & 1 \end{vmatrix}$. 5+5

8. a) Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

b) Find the value of K such that the following system of linear equations is consistant :

$$2x + y - z = 12, \quad x - y - 2z = -3, \quad 3y + 3z = k.$$ 5+5

9. a) Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

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