11. a) Evaluate the limit : $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+3n}\right]$ as an integral.

b) Prove that $\int_0^\infty e^{-x^2}dx=\sqrt{\frac{\pi}{4}}$. 5+5

12. a) Examine the convergence of $\int_0^1 \frac{x^{p-1}}{1-x}dx$

b) Show that $\int_0^{\frac{\pi}{2}} \frac{x^m}{\sin^n x}dx$ is convergent iff $n<1+m$.

5+5

13. a) Prove that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$.

b) Prove that

i) $\gamma(1)=1$.

ii) $\beta\left(\frac{3}{2},\frac{1}{2}\right)=\frac{\pi}{2}$. 5+5

14. Let $f:[a,b]\to\mathbf{R}$, $g:[a,b]\to\mathbf{R}$ be both integrable on [a, b]. Then show that f + g is integrable on [a, b] and $\int_a^b (f+g)=\int_a^b f+\int_a^b g$. 10

Ex/EE/Math/T/112/2019(Old)

BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old)

MATHEMATICS - IF

(50 marks for each Part)

Use a separate Answer-Script for each Part

Time : Three hours | Full Marks : 100

PART - I

Answer ***any five*** questions.

1. State and prove Mean Value Theorem and give its geometrical interpretation with appropriate diagram. 10

2. If $y=e^{a\,\mathrm{Sin}^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(x^2+a^2)y_n=0$ and also find the value of y_n for $x=0$. 10

3. a) If $f(h)=f(0)+hf^{1}(0)+\frac{h^2}{2!}f^{11}(\theta h)$, $0<\theta<1$, find θ, when $h=1$ and $f(x)=(1-x)^{5/2}$.

b) Expand $\log(1+x)$ in power of x in infinite series stating the condition under which the expansion is valid. 5+5

4. Evaluate

i) $\underset{x\to 0}{\mathrm{Lt}}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

[Turn over

ii) $\underset{x\to 0}{\mathrm{Lt}} \dfrac{e^{x}-e^{-x}-2x}{x-\sin x}$ 6+4

5. State the necessary conditions for maximum and minimum with two variables. Find maxima and minima of the function.

$$4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$$ 10

6. a) State and prove Euler's theorem on homogeneous functions in two variables.

b) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \mathrm{Sin}\,2u.$$

7. If v is a function of x and y, then show that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial v^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}$$

where $x = r\cos\theta$ and $y = r\sin\theta$. 10

PART - II

Answer ***any five*** questions.

8. a) A function f is defined on [0, 1] by $f(x) = \begin{cases} x, & x \in [0,1]\cap Q \\ 0, & x \in [0,1]\setminus Q. \end{cases}$ Find $\int_{\underline{0}}^{1} f$ and $\int_{0}^{\overline{1}} f$. Deduce that f is not integrable on [0, 1].

b) A function f is defined on [a, b] by $f(x) = e^x$. Find $\int_{\underline{a}}^{b} f$ and $\int_{a}^{\overline{b}} f$. Deduce that f is integrable on [a, b]. 5+5

9. a) Let $[a,b] \subset R$ and $f:[a,b] \to \mathbf{R}$ be a function of bounded variation on [a, b]. Then prove that f is bounded on [a, b].

b) $f:[0,3] \to \mathbf{R}$ be defined by $f(x) = x^2 - 4x + 3$, $x \in [0,3]$. Show that f is a function of bounded variation on [0, 3]. Calculate $V_0^3(f)$. 5+5

10. a) A function $f:[a,b] \to \mathbf{R}$ is bounded on [a, b] and for every $c \in (a,b)$, f is integrable on [c, b]. Prove that f is integral on [a, b].

b) Prove that $\dfrac{1}{2} < \displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}} < \frac{\pi}{6}$. 5+5