6. (a) Find the complete integral of $p^2 = qz$. 4

(b) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x-y}$$ 8

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x+y)$$ 8

7. A string of length ℓ fixed at both ends is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity given by

$$v = cx, \text{ if } 0 \le x \le \frac{\ell}{2}$$
$$= c(\ell - x) \text{ if } \frac{\ell}{2} \le x \le \ell$$

Find the displacement function y(x,t) of the string. 20

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Ex./CE/MATH/T/211/2019(OLD)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2019

(2nd Year, 1st Semester, Old Syllabus)

Mathematics - III C

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Show that the differential equation

$$\frac{dy}{dx} = \frac{\tan y - y - 2xy}{\sec^2 y - x\tan^2 y + x^2 + 2}$$

is an exact equation and solve it. 5

(b) Solve the differential equation

$$\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$$ 4

(c) State the order and degree of the differential equation :

$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$ 2

(d) Solve :

$$\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{3x} + \cos 5x$$ 9

2. (a) Find the solution of the differential equation in series form near the origin :

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$ 8

(b) Prove that

(i) $n P_n(x) - (2n-1)\, x\, P_{n-1}(x) + (n-1)\, P_{n-2}(x) = 0$

(ii) $x P_n^{/}(x) - P_{n-1}^{/}(x) = n P_n(x)$ where $P_n(x)$ is Legendre polynomial of degree n. 6+6

3. (a) Prove that

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0 \qquad ,(m \neq n)$$

$$= \frac{2}{2n+1} \; ,m = n, n = 0,1,2,...$$ 10

(b) Establish the relation

$$e^{\frac{x}{2}\left(t - \frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$$

where $J_n(x)$ is Bessel function of order n. 10

4. (a) Find the Fourier series for

$f(x) = x^2$ in $-\pi \le x \le \pi$ and deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$$ 12

(b) Find the Fourier sine series for the function

$f(x) = \pi - x$ in $0 < x < \pi$. 8

5. (a) State the convolution theorem for Laplace transform. By using convolution theorem find

$$L^{-1}\left[\frac{1}{\left(S^2+1\right)\left(S^2+4\right)}\right]$$ 10

(b) Solve by the method of Laplace transform the following differential equation :

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 0$$

subject to the conditions

$y = 4$ when $x = 0$

$$\frac{dy}{dx} = -16$$ 10