10. (a) Prove that the straight lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and}$$

$4x - 3y + 1 = 0 = 5x - 3z + 2$, are coplanar.

(b) Find the point where the line joining the points (2,–3,1) and (3,–4,–5) cuts the plane

$$2x + y + z = 7.$$

11. (a) Find the equation of sphere having the circle $x^2 + y^2 + z^2 - 10y + 2z - 8 = 0$, $x + y + z = 2$ as a great circle.

(b) Find the values of c for which the plane x+y+z=c touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$$

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Ex./CE/MATH/T/113/2019(OLD)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old Syllabus)

Mathematics - II C

Time : Three hours Full Marks : 100

Use separate Answer-Scripts for each part.

PART - I

Answer any ***five*** questions.

1. (a) Find the eigen values and corresponding eigen vector of the matrix

$$A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$$

(b) If $A=\begin{pmatrix}1 & 0 & 2\\ 0 & -1 & 1\\ 0 & 1 & 0\end{pmatrix}$, then verify

Cayley Hamilton theorem for the matrix A and hence find A^{-1}. 5+5

2. (a) Solve the system of equations by Cramer's rule : $x + 2y - 3z = 1$, $2x - y + z = 4$, $x + 3y = 5$.

(b) Define eigen values of a matrix show that if λ is an eigen value of a non singular matrix A, then λ^{-1} is also an eigen value of A^{-1}. 5+5

3. (a) Expand by Laplace's method to prove that

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = \left(a^2+b^2+c^2+d^2\right)^2$$

(b) Solve, if possible, the system of equations

$x_1 + 2x_2 - x_3 = 10$

$-x_1 + x_2 + 2x_3 = 2$

$2x_1 + x_2 - 3x_3 = 2$ 5+5

4. (a) Test for convergence of the following series

(i) $\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \frac{11}{4.5.7} + \ldots$

(ii) $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \ldots + \left(\frac{n}{2n+1}\right)^n + \ldots$

(b) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent. 6+4

(b) Find the area of the triangle having the vertices at

$$\vec{P}(1,2,3), \vec{Q}(2,-1,1), \vec{R}(-1,2,3)$$

8. (a) Find out unit vector parallel to the xy plane and perpendicular to the vector $4\hat{i} - 3\hat{j} + \hat{k}$.

(b) Prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$$

9. (a) A plane passing through a fixed point (a,b,c) cuts the axis in A, B, C show that the locus of the center of the sphere OABC is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

(b) Show that the unit vector perpendicular to both the vectors $(3\hat{i} + \hat{j} + 2\hat{k})$ and $(2\hat{i} - 2\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ and the angle between them is $\sin^{-1}\frac{2}{\sqrt{7}}$.

(b) Prove the acute angle between the lines whose direction cosines are given by the relations

$$l+m+n=0 \ and \ l^2+m^2-n^2=0 \ is \ \frac{\pi}{3}$$

8. (a) Prove that the straight lines whose direction cosines are given by

al + bm + cn = 0 and fmn + gnl+ hlm = 0,

are perpendicular if

$$\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0.$$

and parallel if

$$\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0.$$

(b) Find the point where the line joining the points (2,–3,1) and (3,–4,–5) cuts the plane

2x + y + z = 7.

9. (a) Find the equation of the image of the line

$$\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{4}$$

in the plane

3x + y – 4z + 21 = 0

5. (a) Expand θ in powers of tan θ.

(b) If $\frac{Z-i}{Z+1}$ is purely imaginary, then show that the point Z lies on a circle. 5+5

6. (a) If $a+\frac{1}{a}=2\cos\alpha$, then show that

$$a^n+\frac{1}{a^n}=2\cos n\alpha \ , \ a^n-\frac{1}{a^n}=2i\sin n\alpha$$

(b) If Z = Cos + i sin θ and n is a (+)ve integer, then show that

$$(1+z)^n+\left(1+\frac{1}{z}\right)^n=2^{n+1}Cos^n\frac{\theta}{2}+\cos\frac{n\theta}{2}.$$ 5+5

PART - II (50 marks)

Answer any ***five*** questions.

All questions carry equal marks.

7. (a) Find the vector $\vec{d}$ which is perpendicular to both $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}; \vec{b}=\hat{i}-4\hat{j}+5\hat{k} \ and \ \vec{d}.\vec{c}=21$ where

$$\vec{c}=3\hat{i}+\hat{j}-\hat{k}$$

(Turn over)