Ex./CE/MATH/T/112/2019(OLD)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old Syllabus)

Mathematics - I C

Time : Three hour   Full Marks : 100

Answer any ***five*** questions.

1. (a) State Leibnitz theorem. Using this theorem find y_n where $y = e^x \log x$. 5

   (b) If $y = \cos(m \sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$. Find y_n at $x = 0$. 10

   (c) Find y_n where $y = \sin 3x \cos 2x$. 5

2. (a) State and prove Rolle's theorem. Verify Rolle's theorem for the function

   $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1,3]$. 2+5+5

   (b) Examine the validity of Lagrange's Mean-Value theorem for

   $f(x) = x^2 + 3x + 2$ in $[1,2]$. 5

(Turn over)

(c) Evaluate the limit

$$\lim_{x\to 0} \frac{\log(1-x^2)}{\log\cos x}$$ 3

3. (a) Using Maclaurin's theorem, expand f(x) = sin x in a series in powers of x with remainder in Lagrange's form. 10

(b) Find the maximum and minimum values of $f(x) = 2x^3 - 21x^2 + 36x - 20$. 6

(c) Find $\frac{dy}{dx}$ form the following relation :

$e^{xy} - x^2 + y^3 = 0$. 4

4. (a) Find the limit :

$$\lim_{\substack{x\to 1 \\ y\to 2}} \frac{3x^2y}{2x^2+2y^2+1}$$ 3

(b) Discuss the continuity of the function

$$f(x,y) = \frac{xy}{x^2+y^2}\ ,\ (x,y)\neq 0$$
$$= 0,\ (x,y)=0$$

at (0,0). 4

8. (a) Evaluate $\int_0^{12} \frac{1}{1+x}dx$ using Simpson's rule with $h=2$. 8

(b) Using Trapezoidal rule, evaluate $\int_0^1 \sqrt{x^2+1}\,dx$, with 5 subintervals, correct upto 3 decimal places. 12

—— X ——

6. (a) Discuss the convergence of

$$\int_0^{\infty}\left(\frac{1}{1+x}-\frac{1}{e^x}\right)^{\frac{1}{x}}dx$$ 4

(b) Prove that $\frac{\pi^2}{9}<\int\limits_{\pi/6}^{\pi/2}\frac{x}{\sin x}\,dx<\frac{2\pi^2}{9}$. 4

(c) Express the integral $\int_0^1 x^m\left(1-x^w\right)^p dx$ interms of Gamma functions and evaluate $\int_0^1 x^2\left(1-x^2\right)^4 dx$. 12

7. (a) Find the volume of the solid formed by the rotation of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the major axis and rotating about ox. 10

(b) Evaluate $\iint\limits_R\left(x^2+y^2\right)dxdy$ over R where R is bounded by $y=x^2$, $x=2$, $y=1$. 10

(c) Find f_x and f_y where $f(x,y)=\frac{xy}{y-x}$. 4

(d) If $u=\tan^{-1}\frac{xy}{\sqrt{1+x^2+y^2}}$

show that $\frac{\partial^2 u}{\partial x\partial y}=\left(1+x^2+y^2\right)^{-3/2}$. 4

(e) If z = f(x– ct) + g(x– ct) then show that

$$\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}$$

where is a constant. 5

5. (a) State and prove Euler's theorem on homogeneous function. 7

(b) If $u=\sin^{-1}\frac{x^2+y^2}{x+y}$

show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$. 5

(c) Find the dimensions of rectangular box without top with a given volume so that the material used is least. 8

(Turn over)