15. Solve the one dimensional wave equation :

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}, \ 0 \le x \le \ell, \ t > 0,$$

satisfying the boundary conditions : y(0,t) = 0 = y(a,t) for all t. 10

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BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - III C

Time : Three hours Full Marks : 100

Use a separate answer script for each part.

Notations/Symbols have their usual meanings.

PART - I (50 marks)

Answer any ***five*** questions.

1. Solve the following equations :

(a) $\frac{dy}{dx} + x\sin 2y = x\cos^2 y$

(b) $\sin y\frac{dy}{dx} = \cos x\left(2\cos y - \sin^2 x\right)$ 5+5

2. Solve :

(a) $(x-3y+2)dx - (2x-7y+5)dy = 0$

(b) $(D^2 - 1)y = x^2$. 5+5

3. (a) Construct a differential equation by eliminating the parameter λ from the equation

$$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$$

where a and b are fixed constants. 4

(b) Solve :

$$(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=8(5+2x)^2$$ 6

4. Find the series solution of Bessel equation. 10

5. (a) Prove the recurrence formula

$$P_n^{/}(x)=xP_{n-1}^{/}(x)+nP_{n-1}(x)$$

(b) Find the Fourier series for the function

$f(x) = -x, \ -\pi < x < 0$

$\quad = 2x, \ 0 < x < \pi.$ 5+5

6. Find the series solution near x = 0 of the differential equation

$$x^2\frac{d^2y}{dx^2}+\left(x^2+x\right)\frac{dy}{dx}+(x-9)y=0$$ 10

11. Solve :

(a) $\frac{y^2z}{x}p+xzq=y^2$

(b) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ 5+5

12. Solve :

(a) $z = p^2x + q^2y$

(b) $zpq = p + q$ 5+5

13. Determine the solution of the Laplace's equation :

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2z}{\partial x^2}=0,\quad 0<x<a,\quad 0<y<b$$

subject to the following initial and boundary conditions :

$u(0,y) = u(a,y) = 0$ for $0 \le y \le b$ and $u(x,0)=0$, $u(x,b) = f(x)$ for $0 \le x \le a$. 10

14. Solve the one dimensional heat equation :

$$\frac{\partial u}{\partial t}=k\frac{\partial^2u}{\partial x^2},\quad 0<x<\pi,\quad t>0$$

satisfying the following conditions :

$u_x(0,t) = u_x(\pi) = 0$ and $u(x,0) = \sin x$. 10

(b) Let f(z) = u(x,y) + iv(x,y) be an analytic function. Then show that,

(i) u and v are harmonic functions.

(ii) The family of curves u(x,y) = c_1 and v(x,y) = c_2 are orthogonal to each other. 6

OR

(a) Find the general solution of the following partial differential equations :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 3x + 2y$$ 6

(b) Find the complementary function of the following partial differential equations :

$$\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^2 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 0$$ 4

10. (a) Form the partial differential equation by eliminating the arbitrary constants a and b from the following : 6

(i) z = ax + by + ab

(ii) z = $(x^2+a)(y^2+b)$

(b) Solve the following partial differential equation :

$$\frac{\partial^2 z}{\partial x \partial y} = \cos(2x + 3y)$$ 4

7. Prove that

$$J_n(x) J^1_{-n}(x) - J^1_n(x) J_{-n}(x) = -2\frac{\sin(n\pi)}{x\pi}.$$ 10

PART - II (50 marks)

Answer any ***five*** questions.

8. (a) Find Laplace transform of t sin at. 3

(b) Use covolution theorem to find inverse Laplace transform of $\frac{1}{(s+1)(s-1)}$. 3

(c) Use Laplace transform method to solve the following differential equation :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = 0, x(0) = x'(0) = 1$$ 4

9. (a) Prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0$$

If f(z) is an analytic function. 4