

COMPUTATIONAL FLUID DYNAMICS

Time: three hour

Full marks: 100

Answer any four questions

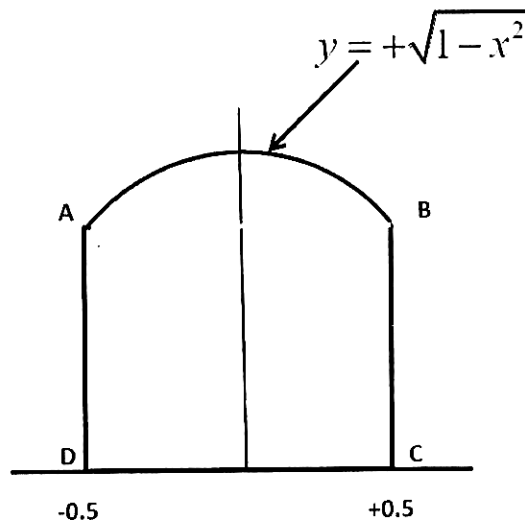
Assume any missing data

All questions carry equal marks

Symbols have usual significance

1. Consider a plane wall of thickness 30 cm whose left surface is at 600 K and right surface is at 400 K. The thermal conductivity of the material is 1.0 W/m.K. Find mid-plane temperature of the wall applying Finite element method. Derive stepwise formulation of the problem. Compare the result with analytical solution.

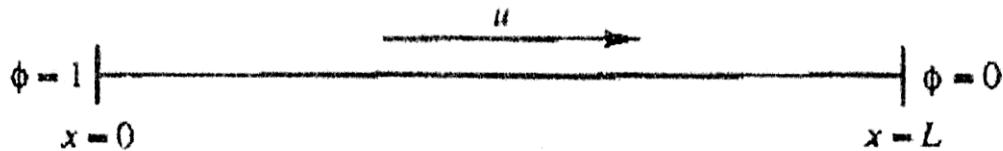
2. Draw grid lines on the associated 2D figure by taking $3 \times 3 = 9$ internal nodes and required number of boundary nodes. Discuss on curvature effect on the grid lines. Show how transformation between physical and computational plane is done. Discuss advantages and disadvantages of solving field equations in physical plane and computational plane.



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3. A plane wall 20 cm thick have left surface temperature 500 K and the right surface temperature 700 K . The volumetric heat generation rate in the wall is 1000 W/m^3 . Using finite volume method, calculate temperature profile in the wall. Data : $k = 0.5 \text{ W/m.K}$ and density = 2000 kg/m^3 . Take three equidistant internal nodes for formulation.

4. a property ϕ is transported by means of convection and diffusion trough one dimensional domain as shown in the following figure.



The governing equations are:

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx} \left(\Gamma \frac{d\phi}{dx} \right)$$

And

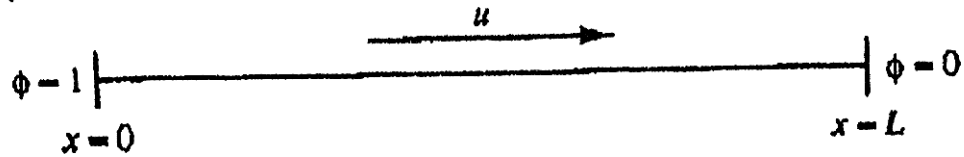
$$\frac{d(\rho u)}{dx} = 0$$

Use finite volume central difference scheme and three equidistant internal nodes to find out distribution of ϕ and compare the computational results with analytical solution given as,

$$\frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\rho u x / \Gamma) - 1}{\exp(\rho u L / \Gamma) - 1}$$

Data: $L = 1 \text{ m}$, $\rho = 1.0 \text{ kg/m}^3$, $\Gamma = 0.1 \text{ kg/m/s}$, and $u = 0.1 \text{ m/s}$.

5. For a 1-D convection diffusion problem, formulate a solution scheme based on finite volume upwind method. Using this method solve following 1-D convection diffusion problem by taking three equidistant nodes.



The governing equations are:

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx} \left(\Gamma \frac{d\phi}{dx} \right)$$

With continuity equation,

$$\frac{d(\rho u)}{dx} = 0$$

Data: $L = 1$ m, $\rho = 1.0$ kg/m³, $\Gamma = 0.1$ kg/m/s, and $u = 3$ m/s.