Bachelor of Engineering in Chemical Engineering Examination, 2019

(2nd Year, 1st Semester, Old)

Mathematics - III B

Time : Three hours Full Marks : 100

Answer ***any five*** questions . 20×5=100

1. a) Let $\sum_{n=1}^{\infty} a_n$ be a series of real numbers. What is meant by the sum of this series ?

   b) What is meant by a geometric series ?

   c) Using comparison test or otherwise prove that the series $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\cdots$ is convergent.

   d) Using D' Alembert's ratio test or otherwise test the convergence of the series

   $$1+\frac{1^2\cdot 2^2}{1\cdot 3\cdot 5}+\frac{1^2\cdot 2^2\cdot 3^2}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\cdots$$ 2+2+6+10

2. a) Find the Fourier series for the function f defined by $f(x)=x-x^2,\ -\pi<x<\pi$. Deduce that

   $$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots=\frac{\pi^2}{12}$$

[Turn over

b) Expand $f(x) = \sin x (0 < x < \pi)$ in cosine series

(10+2)+8

3. a) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xe^x \sin x$

b) Using the method of variation of parameters

solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$. 10+10

4. Find series solution about $x = 0$ of the differential equation

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$$ 20

5. a) Form the partial differential equations from the relation $z = ax + a^2y^2 + b$.

b) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

c) Solve by Charpit's method $px + qy = pq$ where p, q are as in (b). 5+8+7

6. a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions $u(x,0) = 0$, $u(x,a) = 0$, $u(r,y) \to 0$ as $x \to \infty$ when $x \geq 0$ and $0 \leq y \leq a$.

b) Obtain the recurrence relation for the Legendre polynomial $P_n(x)$,

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$ 15+5

7. a) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state condition prevails. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

b) Prove that $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$ where $J_n(x)$ denotes the Besel function. 15+5