13. Prove that

a) $nP_n(x)-(2n-1)xP_{n-1}^{(x)}+(n-1)P_{n-2}^{(x)}=0$

b) $xP_n'(x)-P_{n-1}'(x)=nP_n(x)$

where P_n is Legendre polynomial of degree n. 10

14. a) From partial differential equation from the relations :

i) $xyz=\varphi(x+y+z)$

ii) $z=f(x^2-y^2)$

b) Solve : $\dfrac{\partial^2 z}{\partial x^2}+z=0$

subject to the condition $z=e^y,\ \dfrac{\partial z}{\partial x}=1$

when $x=0$.

15. a) Solve by Charpit's method :

$Z=p^2x+q^2y$

b) Solve by Lagrange's method :

$(mz-ny)p+(nx-lz)q=ly-mx$

where $p=\dfrac{\partial}{\partial x},\ q=\dfrac{\partial}{\partial y}$. 10

Ex/ChE/Math/T/215/2019

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2019

(2nd Year, 1st Semester)

MATHEMATICS - III

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use a separate Answer-Script for each Part

PART - I

Answer question no. 7 and ***any four*** from the rest.

1. a) If L[f] be the Laplace Transform of f(x) prove that

$$L[f''(x)]=s^2L[f(x)]-sf(0)-f'(0).$$

b) If f(x) and $f'(x)$ are Laplace Transformable and if $L[f(x)]=F(s)$ then prove that

$$\lim_{s\to\infty} sF(s)=\lim_{x\to 0} f(x)$$ 12

2. a) Find the Laplace transform of

i) $t^2+\cos 2t\cos t+\sin^2 2t$

ii) $t\,e^{-t}\mathrm{Cost}$ 12

b) Find Inverse Laplace transform of

$$\frac{1+2s}{(s+2)^2(s-1)^2}$$

3. Using Laplace Transform method, solve $\frac{d^2y}{dt^2}+\frac{dy}{dt}=t^2+2t$, given that $y=4$ and $y'=-2$ when $t=0$. 12

4. Find the Fourier series for $f(x)=x^2$ in $-\pi \le x \le \pi$ and hence deduce that

$$\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$ 12

5. a) Find the half range Fourier sine series of $f(x)=x$ in $0<x<2$.

b) Find the Fourier cosine series of $f(x)=\pi-x$ in $(0,\pi)$.

12

6. a) Find the Fourier transform of $f(x)=x\,e^{-x}$ for $0 \le x < \infty$.

b) Show that $f(x)=e^{\frac{-x^2}{2}}$ is self-reciprocal with respect to Fourier transform. 12

7. If $u(n)=k,\ n \ge 0$ be a constant sequence k, k, ... then show that the z-transform of u(n) is $\frac{kz}{z-1}$. 2

PART - II

Answer ***any five*** questions.

8. Solve the following differential equations :

a) $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$

b) $p^3-4xyp+8y^2=0$

c) $y=2p+3p^2$ where $p=\frac{dy}{dx}$ 10

9. Solve : $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2+e^x+\sin 2x$ 10

10. Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$$ 10

11. Find the series solution of

$$(x^2+1)y''+xy'-xy=0$$

about $x=0$. 10

12. find the series solution of the initial value problem :

$xy''+y'+2y=0$

where $y=1,\ y'=2$ at $x=2$

in powers of $(x-1)$. 10