BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester)

MATHEMATICS - II

Time : Three hours Full Marks : 100

Part - I

Answer any four questions. $12.5 \times 4 = 50$

1. (a) Show that the necessary and sufficient condition for a vector function $\vec{F}(t)$ to have direction magnitude is

$$\vec{F}(t) \times \frac{d\vec{F}(t)}{dt} = 0$$

(b) Find the angle between two surfaces

$$xy^2z = 3x + z^2 \;\; and \;\; 3x^2 - y^2 + 2z = 1 \;\; at \; (1, -2, 1).$$

2. Prove that:

$$(i) \; curl \; grad \; \phi = 0 \;\; (ii) \; div \; curl \; \vec{F} = 0$$

3. State Stoke's theorem. Verify Stoke's theorem where

$$\vec{F} = y\,\vec{i} + (x - 2xz)\,\vec{j} - xy\,\vec{k}$$

and the surface S is the part of the sphere

$$x^2 + y^2 + z^2 = a^2$$

above xy plane.

4. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2+y^2)\vec{i} - 2xy\vec{j}$ and the curve C is the rectangle in the xy plane bounded by $y = 0,\ x = a,\ y = b,\ x = 0$.

(b) Find the directional derivative of a scalar point function $f(x, y, z)$ along any line whose direction cosines are l, m, n.

5. State Gauss Divergence theorem. Verify Gauss Divergence theorem for $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2+y^2 = 4$, z=0 and z=3.

6. (a) If $\vec{A}$ and $\vec{B}$ are irrotational , then prove that $\vec{A} \times \vec{B}$ are solenoidal.

(b) Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative field and find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$.

Part - II

Answer any four questions. $12.5 \times 4 = 50$

7. (a) Express

$$\begin{bmatrix} 2 & 3 & -3 \\ 4 & 5 & 6 \\ -5 & 8 & 9 \end{bmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.

(b) Define orthogonal matrix. If A is an orthogonal matrix, show that $|A| = \pm 1$.

8. Find an orthogonal matrix which diagonalize the matrix

$$A = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$$

Also, Diagonalise A.

9. (a) Obtain the equation of the plane through the straight line

$$3x - 4y + 5z - 10 = 0\ ,\quad 2x + 2y - 3z - 4 = 0,$$

and parallel to the line

$$x = 2y = 3z.$$

(b) Prove that the straight lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \ \textit{and}\ 4x - 3y + 1 = 0 = 5x - 3z + 2,$$

are coplanar.

(c) Find the point where the line joining the points $(2, -3, 1)$ and $(3, -4, -5)$ cuts the plane

$$2x + y + z = 7.$$

10.(a) Find the equation of sphere having the circle

$$x^2 + y^2 + z^2 - 10y + 2z - 8 = 0\ ,\quad x + y + z = 2$$

as a great circle.

(b) Find the values of c for which the plane $x + y + z = c$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$$

11. (a) Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \ \textit{and}\ \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

(b) Find the volume of the tetrahedron whose vertices are

$$(0,1,2),\ (3,0,1),\ ,(1,1,1),\ (4,3,2).$$

12. (a) Solve the system of equations by Cramer's rule :

$$x+2y-3z=1\ ,\quad 2x-y+z=4\ ,\quad x+3y=5.$$

(b) Define eigen values of a matrix . Show that if λ is an eigen value of a non singular matrix A , then λ^{-1} is also an eigen value of A^{-1}.

13 (a) If

$$A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix},$$

then show that $A^2-4A-5I_3=0$.

Hence obtain a matrix B such that $AB=I_3$.

(b) Find the eigen values and the corresponding eigen vector of the matrix

$$A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix},$$