b) Determine the surface area of the solid obtained by rotating $y = \sqrt{9 - x^2}$, $-2 \le x \le 2$. 5+5

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old Syllabus)

MATHEMATICS - I

Time : Three hours Full Marks : 100

Answer ***any ten (10)*** questions

1. a) If $y = e^{ax}\text{Sin}bx$, find y_n

   b) Find y_n if $y = \dfrac{x^2 + x - 1}{x^3 + x^2 - 6x}$ 5+5

2. If $y = \sin(m\sin^{-1}x)$, then show that

   i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$

   ii) $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ 10

3. State and prove Rolle's theorem. Give also geometrical interpretation. 10

4. Expand $f(x) = (1 + x)^m$, where m is any real number, in power of x, in infinite series stating the conditions under which the expansion is valid. 10

5. If $v = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

   i) $\dfrac{\partial V}{\partial x} + \dfrac{\partial V}{\partial y} + \dfrac{\partial V}{\partial z} = \dfrac{3}{x + y + z}$

ii) $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=-\frac{3}{(x+y+z)^3}$

6. a) Examine whether $x^{\frac{1}{x}}$ possesses a maxima or a minima, and determine the same.

b) If $\underset{x\to 0}{\text{Lt}}\frac{\text{Sin}2x+a\text{Sin}x}{x^3}$ be finite, find the value of a and the limit. 5+5

7. If v is a function of x and y, prove that

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}$$

where $x=r\cos\theta, y=r\sin\theta$. 10

8. a) A function $f:[0,1]\to R$ is defined by

$$f(x)=\begin{cases}1, & \text{when } x \text{ is rational}\\ 0 & \text{,, } x \text{ ,, irrational,}\end{cases}$$

check whether f Riemann integrable or not. 4+6

b) A function $f:[0,1]\to R$ is defined by f(0) = 0 and $f(x)=1/2^{n-1},\ 1/2^n < x \le 1/2^{n-1},\ n\in N$.

Show that f is Reimann integrable on [0, 1] and find the value of $\int_0^1 f(x)dx$.

9. a) State and prove the fundamental theorem of integral calculus.

b) Evaluate

$$\iint_R \sin(x+y)dxdy \text{ over } R:\{0\le x\le \pi/2, 0\le y\le \pi/2\}$$

6+4

10. a) Test the convergence of $\int_1^\infty \frac{x^2dx}{(1+x^2)^2}$

b) Show that $\int_0^1 \frac{1}{(x+1)(x+2)\sqrt{x(1-x)}}dx$ is convergent.

5+5

11. a) Prove that

$$\int_0^{\pi/2}\sin^p x dx\times\int_0^{\pi/2}\sin^{p+1}x dx=\frac{\pi}{2(p+1)}, p>-1.$$

b) Prove that

$$\int_a^b (x-a)^{m-1}(b-x)^{n-1}dx=(b-a)^{m+n-1}\beta(m,n),$$

$m>0, n>0$. 5+5

12. a) Calculate the following integral by using simpson's 1/3 rule by taking $n=k$

$$\int_1^5 (x^2+1)dx$$