Ref. No. : Ex/ CSE/ T/ 214A/ 2019

B. E. Computer Science and Engineering 2nd Year 1st Semester - 2019

NUMERICAL METHODS

Time: 3 Hrs. **Full Marks: 100**

1. (a) Illustrate **Round-off** and **Truncation** errors in the context of numerical approximations. Calculate the error introduced in a computation due to multiplication of two approximate numbers. 4+4

(b) Describe **Lin's** method for finding the complex roots of a polynomial equation. 12

2 (a) Develop **Multi-point Iteration** formula for solution of non-linear equations. Compute the order of convergence of this method. 14

(b) Find the cube root of 18 correct up to 4 decimal places using **Newton – Raphson** method. 6

OR

(a) Find the condition of convergence and order of convergence for **Single-point iteration** formula for solution of non-linear equations. 8

(b) Find the root of the equation $xe^x - 3 = 0$ that lies between 1 and 2 correct up to 4 decimal places using ***Regula Falsi*** method. Show all the steps in tabular form. 12

3. (a) Show that a $n \times n$ matrix has n different eigenvalues. Hence find all the eigenvalues of the following matrix. 4+4

$$A = \begin{matrix} 5 & 2 & 1 \\ 2 & 5 & 1 \\ 2 & 1 & 5 \end{matrix}$$

(b) Apply **Gauss- Jordan** elimination method to invert the following matrix. Illustrate each step clearly. 12

$$A = \begin{matrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{matrix}$$

Show that $AA^{-1} = I$

OR

(a) Find all the eigenpairs (λ_i, X_i) of the following matrix by **Jacobi's** method. 8

$$A = \begin{matrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{matrix}$$

(b) Sketch **Jacobi's** iterative method for solving linear simultaneous equations using matrix notation. Hence find the condition of convergence. 6+2

Apply **Gauss - Seidel** iterative method to solve the following system of equations. Solution is required correct up to 3 decimal places. 4

[Turn over

$$\begin{aligned} 10x + 2y + z &= 9 \\ x + 10y - z &= -22 \\ -2x + 3y + 10z &= 22 \end{aligned}$$

4. (a) Develop **Modified Euler's** formula for solution of initial value problem of 1st order differential equations. **6**

(b) Formulate the expression for truncation error of the above method. **4**

(c) Prepare a solution table for the following differential equation by **Euler's** method. **10**

$$\frac{dy}{dx} = x^2 + y \qquad \textbf{with } y(0) = 1.0$$

Compute the first **5** steps of the solution with step size $h = 0.2$
Compare the results with those obtained from the exact solution

$$y = 3e^x - x^2 - 2x - 2$$

OR

(a) Develop **Runge- Kutta 4th order** formula for solution of initial value problem of 1st order differential equations. **8**

(b) Explain why this method gives more accurate solution than that of **Euler's** method. **4**

(c) Generalise this method for solution of 2nd order differential equations. **8**

5. (a) Given the following tabular values: **5+5**

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$.
Develop the requisite formulae.

(b) Apply **Romberg's** method to evaluate the following integral correct up to 4 decimal places. Hence obtain the value of π. **10**

$$\int_0^1 \frac{1}{x^2+1}\,dx$$

OR

(a) Given the following table of values: **10**

x	-5	-3	-1	1	3	5
y	5.5	9.1	14.9	22.8	33.3	46

Obtain a **least squares** fit of the following form to the tabular values. Show each step clearly

$$y = a + bx + cx^2$$

(b) Find the expression for total truncation error for **Simpson's ⅓ rd** rule for numerical integration. **10**