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Ref. No.: Ex/CSE/MATH/T/211A/2019

B.CSE, 2ND YR. 1ST SEMS EXAM, 2019

Mathematics - IV

Full Marks:100 Time: Three Hours

(Notations and symbols have their usual meanings.)

Answer question number 1 and any six questions from the rest.

1. Find the radius of convergence and hence find interval of convergence of the following series: (4)

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n(n+1)^3}.$$

2. Write the solution as power of x up to the term of degree 5 of the differential equation (16)

$$(x^2+1)y'' + xy' + xy = 0,$$

together with the initial conditions $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$. Give justification of assuming power series solution about $x = 0$. Write recursion formula. Find radius of convergence of the series.

3. For the Cauchy-Euler differential equation $8x^2y'' + 2xy' + y = 0$, find
   (a) exact solution, (6)
   (b) Frobenius series solution about $x = 0$. (10)

   Compare the series solution with the exact solution.

4. (a) Find general solution of the differential equation (8)

$$y'' + y' - 2y = 6e^{-2x} + 3e^x - 4x^2.$$

   (b) Use the method of variation of parameters to solve (8)

$$y'' + y = \tan x.$$

   Derive the differential equations for the variable parameters. Hence find the solution.

5. (a) Prove that (8)

$$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} t^n P_n(x),$$

   where $|t| < 1, |x| \leq 1$.

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(b) Prove that (8)

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n,$$

where $P_n(x)$ is the Legendra polynomial of degree n. Hence express $P_n(x)$ as a polynomial in x.

6. (a) Find the polynomial solution of the Tchebyshev differential equation (10)

$$(1-x^2)y'' - xy' + n^2y = 0,$$

where n is a constant. For what values of the parameter n the solutions are polynomials?

(b) Show that the above equation can be solved exactly for some suitable transformation of the independent variable x. Distinguish the solution which correspond to Tchebyshev polynomials. (6)

7. (a) Using the definition of limits, show that (4)

$$\lim_{z\to\infty}\frac{1}{z^2} = 0.$$

(b) Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of z. (8)

(c) Find the following limit: (4)

$$\lim_{z\to\infty}\frac{z}{2-iz}.$$

8. **(a) Show that, the function $f(z) = \sqrt{|xy|}$ is not analytic at origin although the** Cauchy-Riemann equations are satisfied at that point. (6)

(b) Evaluate $\int_C (x^2 - iy^2)dz$ along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$. (5)

(c) Expand $f(z) = \frac{1}{z(z^2-3z+2)}$ in a Laurent's series valid for the region $1 < |z| < 2$. Write first five terms of the series. (5)

9. (a) Evaluate $\int_C \frac{1}{z(z-1)}dz$ using Cauchy's integral formula, where C is the circle $|z| = 3$. (6)

(b) What kind of singularity has the function $f(z) = \tan\frac{1}{z}$? Give reasons. (3)

(c) Determine the poles of the function (7)

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$

and find the residue at each pole. Hence evaluate $\int_C f(z)dz$, where C is the circle $|z| = 2.5$.

10. (a) Expand $f(x) = x$ in Fourier series on the interval $-\pi \le x \le \pi$. (6)

(b) Expand Fourier cosine series of the function $f(x) = x\sin x$ on $[0,\pi]$. Hence deduce that (10)

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots$$