Bachelor of Computer Science and Engineering
2^{nd} Year, 2^{nd} Semester Examination, 2019
Graph Theory and Combinatorics

Full Marks: 100 Time : 3 Hr

Section 1 Answer all questions $(5 \times 5 = 25)$

(a) Explain with examples of the following $(3 \times 1 + 2 = 5)$

(i) Pendant vertex
(ii) Null graph
(iii) Forest
(iv) Kurotowski's graphs

(b) Is $4, 4, 3, 2, 2, 1, 1$ graphical? If not, explain why; if so, find a simple graph with this degree sequence.

(c) Prove that the sum of the degrees of a graph is twice the number of edges in it.

(d) A **bit** is either 0 or 1; a **byte** is a sequence of 8 bits. Find

(i) number of bytes (0.5)
(ii) the number of bytes that begin with 11 and end with 11 (0.5)
(iii) the number of bytes that begin with 11 and do not end with 11 (2)
(iv) the number of bytes that begin with 11 or end end with 11 (2)

(e) There are 15 married couples in a party. Find the number of ways of choosing a woman and a man from the party such that the two are (i) married to each other, (ii) not married to each other.

Section 2 Answer **Any Three** questions $(3 \times 10 = 30)$

(a) (i) Using techniques from Graph theory, show that $1+2+3+\ldots+n = \frac{n(n+1)}{2}$
(ii) Show that the number of terminal vertices in a binary tree with n vertices is $\frac{(n+1)}{2}$.

(b) Obtain a de Bruijn sequence such that any three-letter word using 0, 1 and 2 can be obtained from this sequence.

(c) Suppose there are many red socks, white socks and blue socks in a box.

(i) What is the least number of socks that one should grab from the box (without looking at the contents) to be sure of getting a matching pair?

[Turn over

(ii) Rework the problem if 3 pairs, all of one color, are desired ?

(d) (i) Prove that every fifth Fibonacci number is a multiple of 5

(ii) Solve $a_n = a_{n-2} + 4n$ by use of the characteristic equation

Section 3 Answer any **Two question** $(2 \times 15 = 30)$

(a) (i) Show that a simple graph with three or more vertices is bipartite if and only if it has no odd cycles. (7)

(ii) Consider a bidirectional random walk on the X-axis. The particle starts (at time $t = 0$) from the origin and can make steps of +1 (with fixed probability p) or -1 (with fixed probability $q = 1 - p$). Show that $P_n(r)$ - the probability that the particle is at $x = r$ after n steps – is the coefficient of x^r in the binomial expansion of $(px + \frac{q}{x})^n$ (8)

(b) (i) If throwing a die 5 times constitutes a *trial*, with the 5 throws considered distinguishable, find the number of trials that produce a total of 12 or fewer dots. (8)

(ii) Show by using Kruskal's Algorithm that the network in Figure 1 is a disconnected weighted graph (7)

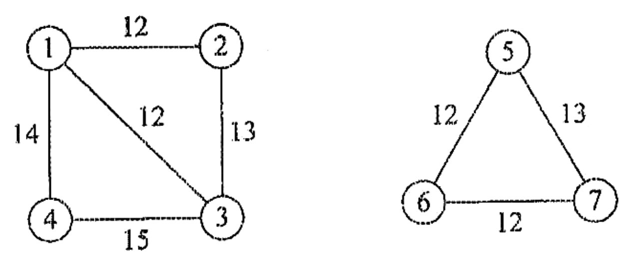

Figure 1: Question for 3(b)ii

(c) (i) Show that the Peterson graph is non-planar by establishing that it has a K-subgraph (5)

(ii) During a month with 30 days, a football team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (5)

(iii) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there? (5)

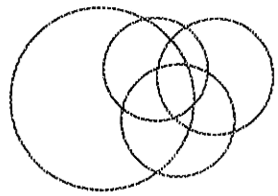

Figure 2: Question 4a

Section 4 Answer any one question (15)

(a) Determine the number h_n of regions that are created by n mutually overlapping circles in general position in the plane as shown in Figure 2. By *mutually overlapping*, we mean that each two circles intersect in two distinct points (thus non-intersecting or tangent circles are not allowed). By *general position*, we mean that there do not exist three circles with a common point.

(b) Show that the maximum number of edges in a bipartite graph on $|V|$ vertices is $[\frac{|V|^2}{4}]$.
Note : $[x]$: *The greatest Integer not greater than the real number x*