Bachelor of Computer Science & Engineering Supplementary Examination 2019 (Old)

(First Year, First Semester)

MATHEMATICS - I

Time : Three Hours   Full Marks : 100

The figures in the margin indicate full marks

Answer Q. No. 9 and any **six** questions from Q. Nos. 1 – 8.

1. (a) Let A, B, C be three subsets of a set X. Show that $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$. 8

   (b) Define *reflexive* and *anti-symmetric relations* on a nonempty set. Let S be a set with 20 elements. Find the number of reflexive and anti-symmetric relations that can be defined on S. 8

2. (a) Define a *permutation* and an *even permutation*. Find the number of even permutations in S_{25}. 8

   (b) Let $\alpha = (2\ 5\ 7\ 9\ 4)(3\ 1\ 8)$, $\beta = (5\ 1\ 7\ 6)(2\ 8\ 6)$ and $\gamma = (4\ 6\ 9\ 5\ 8\ 7)(3\ 2\ 1)$ in S_9. Express $\alpha^3 \beta^{-3} \gamma^3$ as a product of disjoint cycles. 8

3. (a) Define a *countable* set. Prove that a countable union of countable sets is countable. 8

   (b) Prove that $|A| < |\mathscr{P}(A)|$ for any set A, where $\mathscr{P}(A)$ is the power set of A. 8

4. (a) What is a *tautology*? Prove that $p \vee \neg(p \wedge q)$ is tautology. 8

   (b) Let $A = \{1, 2, \ldots, 10\}$. Consider each of the following sentences. If it is a statement, then determine its truth value. If it a proposional function, determine its truth set.

      (i) $(\forall x \in A)(\exists y \in A)(x + y < 4)$. 4

      (ii) $(\forall x \in A)(\forall y \in A)(x + y < 4)$. 4

5. (a) If α, β, γ are the direction cosines of a line, then show that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. 8

   (b) If $\vec{\alpha} = 2\vec{i} - 10\vec{j} + 2\vec{k}$, $\vec{\beta} = 3\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{\gamma} = 2\vec{i} + \vec{j} + 3\vec{k}$. Find the vector $\vec{\alpha} \times (\vec{\beta} \times \vec{\gamma})$ and interpret the result geometrically. 8

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6. (a) Find the equation of the plane which passes through the point $(2, -1, 1)$ and is orthogonal to each of the planes $x - y + z = 1$ and $3x + 4y - 2z = 0$. 8

   (b) Show that the vectors $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} - 4\vec{k}$ form the sides of a right angled triangle. 8

7. (a) Show that the spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$ touch each other. Find there point of contact. 8

   (b) Find the equation of the right circular cone whose vertex is at the origin, axis is $\frac{x}{3} = \frac{y}{2} = \frac{z}{4}$ and semivertical angle is $45°$. 8

8. (a) If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three unit vectors satisfying $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}$, where $\vec{0}$ is the zero-vector, then find $\vec{\alpha} \cdot \vec{\beta} + \vec{\beta} \cdot \vec{\gamma} + \vec{\gamma} \cdot \vec{\alpha}$. 8

   (b) Find the shortest distance between two skew lines $\vec{r} = \vec{r_1} + t\vec{\alpha}$, $\vec{r} = \vec{r_2} + t\vec{\beta}$, where t is a scalar and $\vec{r_1}, \vec{\alpha}, \vec{r_2}, \vec{\beta}$ are the vectors with coordinates $(1, -2, 3)$, $(2, 1, 1)$, $(-2, 2, -1)$, $(-3, 1, 2)$ respectively. 8

9. Let $S = \{n \in \mathbb{N} \mid n \text{ divides } 216\}$. Define a *lattice*. Show that $(S, \vee, \wedge)$ is a lattice, where $a \wedge b = \gcd\{a, b\}$ and $a \vee b = \text{lcm}\ \{a, b\}$. 4