Bachelor of Engineering in Computer Science Engineering Examination, 2019

(1st Year, 2nd Semester)

Mathematics - III

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each group.

PART - I

Answer ***any five*** questions :

1. a) Define group Lat $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0E/R \right\}$. Does G form a commnlative group w.r.t matrix multiplication ? Justify your answer. 2+3

   b) i) Let G be a group in a such the order of each non identify emement be 2. Show that G is a communlative group.

   ii) Let Q be a group and a, b, & G such that 0 (a) =5 and $a^3b=ba^3$. Show that ab=ba. 2+3

2. a) Define centre of a group. Show that the centre of a group G is a subgroup of G. If a is the only elemen of order 2 in a group G then show that a EZ(G) (centre of G). 1+2+2

   b) Define cycticgroup. Show that (Q, t) is not a cyclim

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group. Hence conclude that (R,t) is not a cyclic group. 1+2+2

3. a) State Lagrangels Theoem of group. Is the convense of Lagrange is Theorem hold ? Justify your answer. 2+3

b) Define normal subgroup of a group. Show that SL(2, /R) is a normal subgroup of GL (2,/R) 2+3

4. a) State first isomirphism theorem of group. Show that anyepimorphism from (D, t) onto itself is an isomorphism. 2+3

b) Define Quotiemtgroup. Show that $D/_{nZ} \cong D_n$ 2+3

5. a) Define ring. Show that anyring of prime order in commntative. 2+3

b) Define integral domain and fiels. Shoe that every finite integral domain is a field. 2+3

6. a) Define ideal of a ring. Show that a fiedl E has only two ideals $\{0\}$ and F 2+3

b) Define prime ideal and maximal ideal of an commntativ ring R. Show that {x} is a maximal ideal of Q [x] but [x] is a maximal ideal of Q[x] 5+5

7. a) Define Principle ideal domain (PID). Show that every Enclidean Domain (ED) is a principal ideal domain (PID).

b) Define aljebraic field extension. Show that every finite field extension is an algebraic field extension. Is Q^{N_2}/Q an algebraic field extension ? Justify you answer. 2+2+1

The figures in the margin indicate full marks.

PART - II

Answer ***any five*** questions 10×5

(All question carry equal marks)

8. a) Evaluate the detueminant

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} \text{ where } 2s = a+b+c$$

b) Show that

$$\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(bc+ca+ab)^3$$

5+5

9. a) Define symmetric and skew symmetric diterminants. Prove that a skew -symmetric diterminant of order 4 is a perfect square.

b) Solve the following system of equations using Cramir's rule :

$$\begin{aligned} 3x + y + 2z &= 3 \\ x - 5y - z &= 6 \\ 2x + 3y + 2z &= 0 \end{aligned}$$

4+6

10. a) If

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} \text{verify that } (AB)^{-1} = B^{-1}A^{-1}$$

b) Determine the rank of the following matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

5+5

11. a) Find the eigen volums and corresponding eigen vector of the following matrix :

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

b) State and prove caylay Hamilton Theorem. 6+4

12. a) Prove that the intersection of two subspaces of a vector space V over a field F is a subspace of V, but the union of two subspaces of V is not, in general a subspace of V.

b) Let S be a finite set of vectors in a vector space V over a field F. Show that the set of all linear combination of the vectors in S forms a subspace of V and this is the smallest subspace containing S. 5+5

13. a) Define basis and demension of a vector space with an example.

Let . $S = \{(x, y, z) E\ \mathbb{R}^3 \mid x + y + z = 0\}$ Prove that S is a subspace of $\mathbb{R}^3$. Find a basis and dimension of S.

b) State and prove Deletion Theorem in relation with a vector space. 4+6

14. a) Let U and W be two subspaces of a finite dimensional vector space V our a field F. When prove that

$$\dim(U + W) = \dim(U) + \dim(W) - \dim(U \cap W)$$

b) Define complement of a subspace U in a vector space V. Let U=L{(1, 2, 1), (2, 1, 3)}. Find two different complements of U in $\mathbb{R}^3$ 6+4