BACHELOR OF ENGINEERING IN COMPUTER SCIENCE & ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old)

MATHEMATICS - II

Time : Three hours | Full Marks : 100

Answer ***any five*** questions .

1. a) Define limit of a sequence. Prove that a convergent sequence determines its limit uniquely. 10

   b) Show that the sequence $\{x_n\}_{n\in N,}$ where

$$x_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)}$$

   is monotonically increasing and bounded. 10

2. a) State Cauchy's general principle of convergence of an infinite series. Prove that if $u_n > 0$ and if $\underset{n\to\infty}{\mathrm{Lt}} (u_n)^{\frac{1}{n}} = \rho$ then

   i) Σu_n Converges of $\rho < 1$

   ii) diverges if $\rho > 1$ 10

   b) Determine the radius of convergence and interval of convergence of the series

$$x + \frac{(2!)^2}{4!}x^2 + \frac{(3!)^2}{6!}x^3 + \cdots + \frac{(n!)^2}{(2n)!}x^n + \cdots \quad 10$$

[Turn over

3. a) Show that the function

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & x^2+y^2\neq 0\\ 0, & x=0=y\end{cases}$$

is continuous at (0, 0). 10

b) If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u. \quad 10$$

4. a) State and prove Leibnitz's theorem of nth derivative of the product of two functions. 10

b) Find the value of y_n for $x=0$ when

$y=\cos(m\sin^{-1}x)$ 10

5. a) Expand $\log(1+x)$ stating the condition under which the expansion is valid. 10

b) Show that $\dfrac{x}{1+x}<\log(1+x)<x$ if $x>0$. 10

6. a) State and prove fundamental theorem of integral calculus. 10

b) Show that every continuous function is R-integrable. 10

7. a) Evaluate :

i) $\displaystyle \operatorname*{Lt}_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

ii) $\displaystyle \operatorname*{Lt}_{x\to 0}\left(\frac{x-\sin x}{x^3}\right)$ 6

b) Using the definition of Beta function, prove that

$$\int_0^{\pi/2}\cos^4 x\,dx=\frac{3\pi}{16} \quad 4$$

c) Find the equation of the circle of curvature at the point (3,1) on the curve $y=x^2-6x+10.$ 10