Ex./ARCH/MATH/T/216/2019(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION, 2019

(2nd Year, 1st Semester, Old Syllabus)

Mathematics - III A

Time : Three hours | Full Marks : 100

PART - I (50 marks)

Answer any ***five*** questions.

1. (a) Define the following :
   (i) Mutually exclusive events,
   (ii) Mutually independent events,
   (iii) An exhaustive set of events.

   (b) If A and B are independent events and $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$, then find (i) $P(A\cup B)$ (ii) $P(A^C\cap B)$.

   (3x2)+4

2. (a) State and prove the law of addition of probability for any two events.

   (b) Two players A and B toss a die alternately. He who first throws a 'six' wins the game. If A begins the game, what is the probability of B'swin ? (1+5)+4

(Turn over)

3. (a) A packet of 10 electronic components is known to include 3 defectives. If 4 components are randomly chosen and tested, what is the probability of finding among them not more than one defective ?

(b) Three lots contain respectively 10%, 20% and 25% defective articles. One article is drawn at random from each lot. What is the probability that among them there is (i) exactly one defective, (ii) at least one defective ? $5+(2\text{x}2^1/_2)$

4. (a) Find the mean and variance of the Binomial distribution. State the condition for maximum variance.

(b) A point P is chosen at random on a line segment AB = 2a. O is the middle point of AB. What is the probability that the three line segments AP, PB and AO can form the sides of a triangle ? 5+5

5. (a) Define the following :

(i) Celestial meridian, (ii) Ecliptic, (iii) The first point of Aries.

(b) If the declination and zenith distance of a star on the prime vertical are known then what is the latitude of the place of the observation ? (2x3)+4

13. (a) Using the method of variation of parameters solve the differential equation :

$$\frac{d^2z}{dx^2} - 2\frac{dz}{dx} - 3x = xe^{-x}$$

(b) A population consists of the four members 3, 7, 11, 15 consider all possible samples of size two which can be drawn with replacement from this population. Find the population mean and population standard deviation. 5+5

14. In simple random sampling without replacement, show that the variance of sample mean $\bar{x}$ is given by

$$Var(\bar{x}) = \frac{\sigma N^2 - 1}{n} \cdot \left(\frac{N-n}{n}\right)$$

where $\sigma^2 N-1$ is the population mean square, N and n are the population size and the sample size respectively. 10

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10. (a) If x^{α} be an integrating factor of

$$(x-y^2)dx + 2xy\,dy = 0$$

Then find α and hence solve it.

(b) Solve the Bernoulli's equation

$$xy - \frac{dy}{dx} = y^3 e^{-x^2}$$

5+5

11. (a) Find the general solution of the differential equation :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$$

(b) $(D+2)(D-1)^3 y = e^x$, $D \equiv \dfrac{d}{dx}$. 5+5

12. (a) Solve : $\dfrac{dy}{dx} = \sin(x+2y)$

(b) A sample random sample of size 10 is drawn without replacement from a finite population consisting of 50 units. If the population standard deviation is 36, what is the standard error of sample mean ? 5+5

6. State the essential properties of a spherical triangle. Also write down the sine, cosine and four parts formule in spherical trigonometry. 3+(2+2+3)

7. Draw a celestial sphere, indicating the position of a star. Also show in the diagram the R. A., declination, zenith distance and the latitude of the place. 2+(4x2)

PART - II (50 marks)

Answer any ***five*** questions.

8. (a) When a differential equation is called an exact differential equation ?

(b) Show that the differential equation

$$ydx + (x^2y - x)dy = 0$$

is not exact. Find an integrating factor and hence solve the differential equation. 2+8

9. (a) Solve : $\dfrac{d^3y}{dx^3} + 3\dfrac{d^2y}{dx^2} - 4y = xe^{-2x}$

(b) Solve : $x\cos x \dfrac{dy}{dx} + y(x\sin x + \cos x) = 1$ 5+5