Ex/ARCH/MATH/T/124/2019

BACHELOR OF ARCHITECTURE ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester)

Mathematics - II

Time : Three hours | Full Marks : 100

Use a separate Answer Script for each part.

PART - I (50 marks)

Answer any ***five*** questions.

1. (a) Find the matrix A if adj $A=\begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and det A = 2. 5

(b) (i) Suppose that A and B be two third order matrices such that they differ only in their second column. Show that det(A+B) = 4 [det A + det B].

(ii) Show that the determinant

$$\begin{vmatrix} 2 & 6 & 9 & 7 \\ 2 & 7 & 5 & 9 \\ 2 & 8 & 2 & 1 \\ 2 & 8 & 8 & 3 \end{vmatrix}$$ is divisible by 31. 3+2

(Turn Over)

2. (a) (i) Show that the determinant value of a real orthogonal matrix is ± 1.

   (ii) Let A be a square matrix of order n such that det A = 2. Find det $(A^{-1} + \text{adj } A)$. 2+3

   (b) Let A and B be two real orthogonal matrices of the same order n, where n is odd, such that det A = det B. Show that (A–B) is a singular matrix. 5

3. (a) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$ 5

   (b) Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$ 5

4. (a) (i) If for given A and B, the matrix equation AX = B has more than one solution then show that it has infinitely many solutions.

PART - II (50 marks)

Answer any ***five*** questions.

8. (a) Find the angle between the lines whose direction cosines are proportional to 1, 2, 1 and 2, –3, 6.

   (b) Show that the straight lines whose direction cosines are given by $2\ell + 2m - n = 0$, $mn + nl + lm = 0$ are at right angles. 4+6

9. (a) A, B, C are three points on the axis of x, y and z respectively at distances a, b, c from the origin 0. Find the coordinates of the point which is equidistant from A, B, C and O.

   (b) Find the equation of the plane passing through the line of intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and perpendicular to the plane $2y-3x=4$. 4+6

10. (a) Find the equation of the plane through (1,2,3) and parallel to the plane $4x + 5y - 3z = 7$.

   (b) A plane meets the coordinate axes at A, B, C such that the centroid of the traingle ABC is the point (a,b,c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$. 5+5

(Turn Over)

(ii) If the system of equations $ax+by+cz=0$, $bx+cy+az=0$, $cx+ay+bz=0$ have a non zero solution then show that either $a+b+c=0$ or $a=b=c$. 3+2

(b) If $A=\begin{pmatrix}1 & 2 & 1\\ 1 & -4 & 1\\ 3 & 0 & -3\end{pmatrix}$ and $B=\begin{pmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{pmatrix}$ then show that $AB=6I_3$. Utilize this result to solve the following system of equations

$$2x+y+z=5$$
$$x-y=0$$
$$2x+y-z=1$$
2+3

5. (a) A manufacturer makes red and blue pen. A red pen takes twice as much time as to make a blue pen. If the manufacturer makes only blue pens, 500 can be made in a day. A red pen sells for Rs. 8 and atmost 150 can be sold in a day. A blue pen sells for Rs. 5 and atmost 250 can be sold in a day. The manufacturer desires to maximize his revenue. Formulate the above problem as a linear programming problem. 5

(b) Solve graphically the following linear programming problem :

(Turn Over)

Maximize $Z = 5x_1 + 7x_2$

Subject to $3x_1 + 8x_2 \le 12$

$x_1 + x_2 \le 2$

$2x_1 \le 3$

$x_1, x_2 \ge 0$. 5

6. (a) If $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$ then show that

$$(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = P^{\frac{m}{m-1}}$$
5

(b) If two curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ cut orthogonally then prove that

$$\frac{1}{b_1} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{a}.$$
5

7. (a) Find the radius of curvature at the origin for the curve $x^3 + y^3 - 2x + 6y = 0$. 5

(b) Determine the asymptotes of $x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$. 5

11. (a) Find the equation of the plane which contains the line $\frac{x-1}{2}=\frac{y-3}{1}=\frac{x-13}{2}$ and is perpendicular to the plane x+y+z=3.

(b) Find the equation of the line through (1,2,–1) and perpendicular to each of the lines

$$\frac{x}{1}=\frac{y}{0}=\frac{z}{-1} \textit{ and } \frac{x}{3}=\frac{y}{4}=\frac{z}{5}$$ 5+5

12. (a) Find the equation of the line x+y+z–1=0, 2x–y–3z+1=0 in symmetrical form.

(b) Find the condition that the two lines $\frac{x-x_1}{\ell_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{\ell_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar. 6+4

13. (a) Find the equation of the line of shortest distance between the lines

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

(b) Find the equation of a sphere which panes through (0,0,0), (a,0,0), (0,b,0) and (0,0,c). 5+5

14. (a) Show that the general equation of the cone of second degree which panes through the coordinate axes is of the form

$$fyz + gzx + hxy = 0$$

(b) Prove that the circles

$x^2+y^2+z^2-2x+3y+4z-5=0,\ 5y+6z+1=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0,\ x+2y-7z=0$ lie on the same sphere and find its equation. 4+6

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