11. (a) Prove that $\int_0^{\pi/2} \log\tan x \, dx = 0$

(b) $\int_0^1 \frac{\log x \, dx}{\sqrt{(1-x^2)2}} = \pi/2 \log\frac{1}{2}$ 5+5

12. (a) Show that $\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}\ \Gamma(2n+1)}{2^{2n}\ \Gamma(n+1)}$

(b) Show that

$$\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$$ 5+5

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Ex./ARCH/MATH/T/115/2019(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION, 2019

(1st Year, 1st Semester, Old Syllabus)

Mathematics - I A

Time : Three hours Full Marks : 100

Answer any ***ten*** questions.

1. (a) Find y_n when y = sin ax

(b) If $y = e^{ax} \cos bx$, find y_n. 4+6

2. (a) State and prove Leibritz's theorem.

(b) If $y = e^{a\sin^{-1}x}$, show that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$ 5+5

3. (a) State Rolle's theorem. Give its geometrical interpretation with appropriate figure.

(b) If $f(h) = f(0) + hf^{/}(0) + \frac{h^2}{2!} f^{//}(\theta h), 0 < \theta < 1,$

find θ, when h = 7 and $f(x) = \frac{1}{1+x}$. 5+5

(Turn over)

4. (a) Verify Rolle's theorem for the function $f(x) = |x|$ in the interval $[-1,1]$.

(b) Expand $\sin^3 x$ in the neighbourhood of $x = 0$ to three terms plus remainder in Lagrange's form. 5+5

5. (a) Show that $\sin^3 x \cos x$ is a maximum when $x = \pi/3$. Also find maximum value.

(b) Find minimum and maximum values of $x^3 + 3x^2 y - 15x^2 - 15y^2 + 72x$. 5+5

6. Evaluate

(a) $\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$

(b) $\lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$ 5+5

7. (a) If $u = f(r)$, where $x^2 + y^2 + r^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{f'(r)}{r}$$

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^3}$$ 5+5

8. (a) State and prove Euler's theorem for homogeneous equation for two variables.

(b) If $u = e^{(x^2 + y^2)}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \log u$$ 5+5

9. (a) Prove that

$$\int_0^1 \frac{x^6\, dx}{\sqrt{1-x^2}} = \frac{5\pi}{32}$$

(b) $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{n+r}{n^2 + r^2}$ 5+5

10. (a) Prove that $\int_0^{\pi/2} \frac{dx}{4 + 5\sin x} = \frac{1}{3} \log 2$

(b) Evaluate $\int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ 5+5

(Turn over)