BACHELOR OF ARCHITECTURE EXAMINATION, 2019

(1st Year, 1st Semester)

Mathematics - I

Time : Three hours Full Marks : 100

Use a separate Answer-Script for each part.

PART - I

Answer any ***five*** questions.

1. (a) Find y_n, where y = log(x+a)

   (b) If $y = \dfrac{x^2+x-1}{x^3+x^2-6x}$, find y_n 5+5

2. (a) If $u = \dfrac{y}{z} + \dfrac{z}{x} + \dfrac{x}{y}$, prove that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial y}$$

   (b) Show that

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ if } u = \log(x^2+y^2)$$ 5+5

BACHELOR OF CIVIL ENGINEERING (EVENING) EXAMINATION, 2019

(2nd Year, 1st Semester)

(Turn over)

3. If $y = sin\left(m\sin^{-1}x\right)$, then show that

(a) $(1-x^2)y^2 - xy_1 + m^2y = 0$

(b) $(i-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. Also find $y_n(0)$. 10

4. State and prove Mean Value Theorem. Give its geomerical interpretation with appropriate figure. 10

5. State the necessary and sufficient conditions for maxima and minima. Find the maximum and minimum values of $u = \frac{4}{x} + \frac{36}{y}$ *and* $x + y = 2$. 10

6. (a) If $f(h) = f(0) + hf^{/}(0) + \frac{h^2}{2!} f^{//}(\theta h), 0 < \theta < 1$, find θ, when h = 7 and $f(x) = \frac{1}{1+x}$

(b) Expand the following functions in powers of x in infinite series stating in each case the conditions under which the expansion is valid.

(i) cos x (ii) e^x. 5+5

11. Using Beta Gamma functions, evaluate the following integrals :

$$\int_0^{\pi/2} \sin x^p dx \times \int_0^{\pi/2} \sin x^{p+1} dx, \int_0^{\infty} e^{-x^4} dx \times \int_0^{\pi/2} e^{-x^4} x^2 dx$$

5+5

12. Solve the following integrals by the method of integration by partial fractions :

$$\int \frac{x^2 - x + 1}{x + 1^3} dx, \int \frac{x^4 + x^3 + x^2 + 1}{x^3 + x - 2} dx$$

5+5

13. Calculate the following integrals by using Simpons 1/3 rule by taking n=4 :

(a) $\int_0^1 2x dx$

(b) $\int_1^5 \left(x^2 + 1\right) dx$

14. (a) Determine the surface area of the solid obtain by rotating $y = \sqrt{9 - x^2}, -2 \le x \le 2$ about x-axis.

(b) Determine the surface area of the solid obtain by rotating $y = x^{1/3}, -1 \le x \le 1$ about y-axis. 5+5

—— X ——

(b) A function $f : [-1, 3] \to R$ is defined by $f(x) = [x]$, $x \in [-1,3]$. Show that f is integrable on $[-1,3]$ and calculate $\int_{-1}^{3} f(x)dx$. 4

10. (a) Define antiderivative of a function f on an interval I. 2

(b) A function $f : [-3,3] \to R$ is defined by

$$f(x) = \begin{cases} 2x\sin(\pi/x) + \pi\cos\pi/x, & when\, x \neq 0, \\ 0 & , \quad when\, x = 0. \end{cases}$$

Show that f is Riemann integrable on $[-3,3]$. Using Fundamental Theorem of Integral Calculus evaluate $\int_{-3}^{3} f(x)dx$. 4

(c) A function $f : [-2,2] \to R$ is defined by

$$f(x) = \begin{cases} 3x^2\cos\pi/x^2 + 2\pi\sin\pi/x^2, & when\, x \neq 0, \\ 0 & , \quad when\, x = 0. \end{cases}$$

Show that f is Riemann integrable on $[-2,2]$. Using Fundamental Theorem of Integral Calculus, evaluate $\int_{-2}^{2} f(x)dx$. 4

7. Evaluate

(i) $\lim_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

(ii) $\lim_{x \to 0} (\text{Cos}\, x)^{\cot^2 x}$ 10

PART - II

Answer any ***five*** questions.

8. (a) A function $f : [0,1] \to R$ is defined by

$$f(x) = \begin{cases} 1, when\, x\, is\, rational, \\ 0, when\, x\, is\, irrational \end{cases}$$

Check whether f is Riemann integrable or not. 4

(b) A function $f : [0,1] \to R$ is defined by $f(0)=0$, $f(x) = 1/2^{(n-1)}$, $1/2^n < x \le 1/2^{(n-1)}$, $n \in N$. Show that f is Riemann integrable on $[0,1]$ and find the value of $\int_0^1 f(x)dx$. 3+3

9. (a) State and prove Fundamental Theorem of Integral Calculas. 1+5