ELEMENTS OF COMPUTATIONAL FLUID DYNAMICS

Chapter - 1

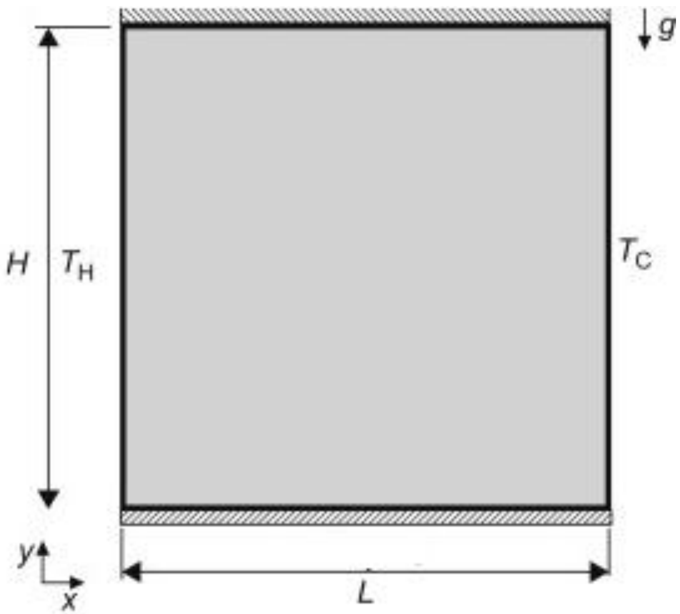
Basic Approach in Solving a Problem by CFD Techniques

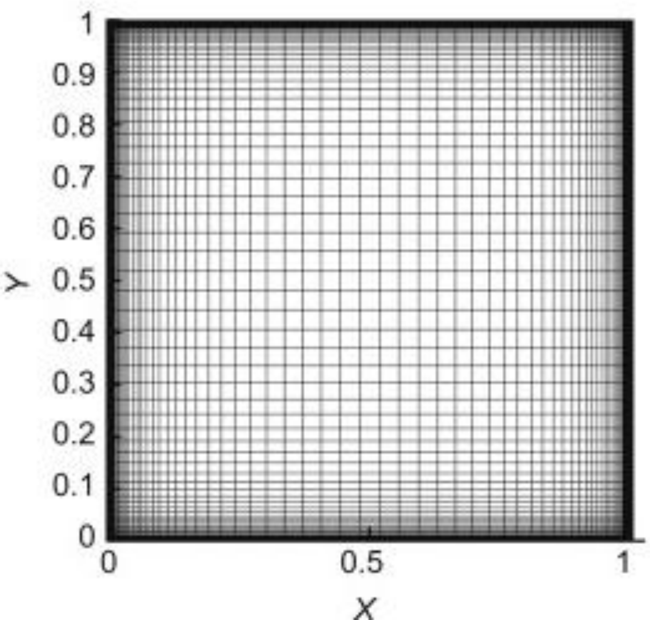
Name of the Approach	Process
Mathematical model	A set of partial differential equations and boundary conditions which include simplifications of exact conservation laws. A solution methodology is usually proposed for a particular set of equations.
Discretization scheme	A suitable discretization scheme is selected such as finite difference, finite element, or finite volume using which the differential equations are approximated by a system of algebraic equations for the variables at some set of discrete points in space and time
Coordinate system	A relevant coordinate system such as Cartesian, cylindrical, spherical, curvilinear orthogonal, or non-orthogonal, which may be fixed or moving, is chosen depending on the problem
Numerical grid	The computational domain is imagined to be filled with a grid that is a network of lines. The intersection of grid lines is called a grid point in the finite-difference scheme. A grid divides the solution domain into a number of subdomains. The conservation equations are assumed to be valid at each grid point in the computational domain in FDM. The position of a grid point is identified by a set of two such as (i, j) in 2D or three such as (i, j, k)in 3D indices. Each point has four surrounding neighbors in 2D and six in 3D. For an unsteady or a time-dependent problem, the additional indices are p and p + 1, indicating the present and future times, respectively.

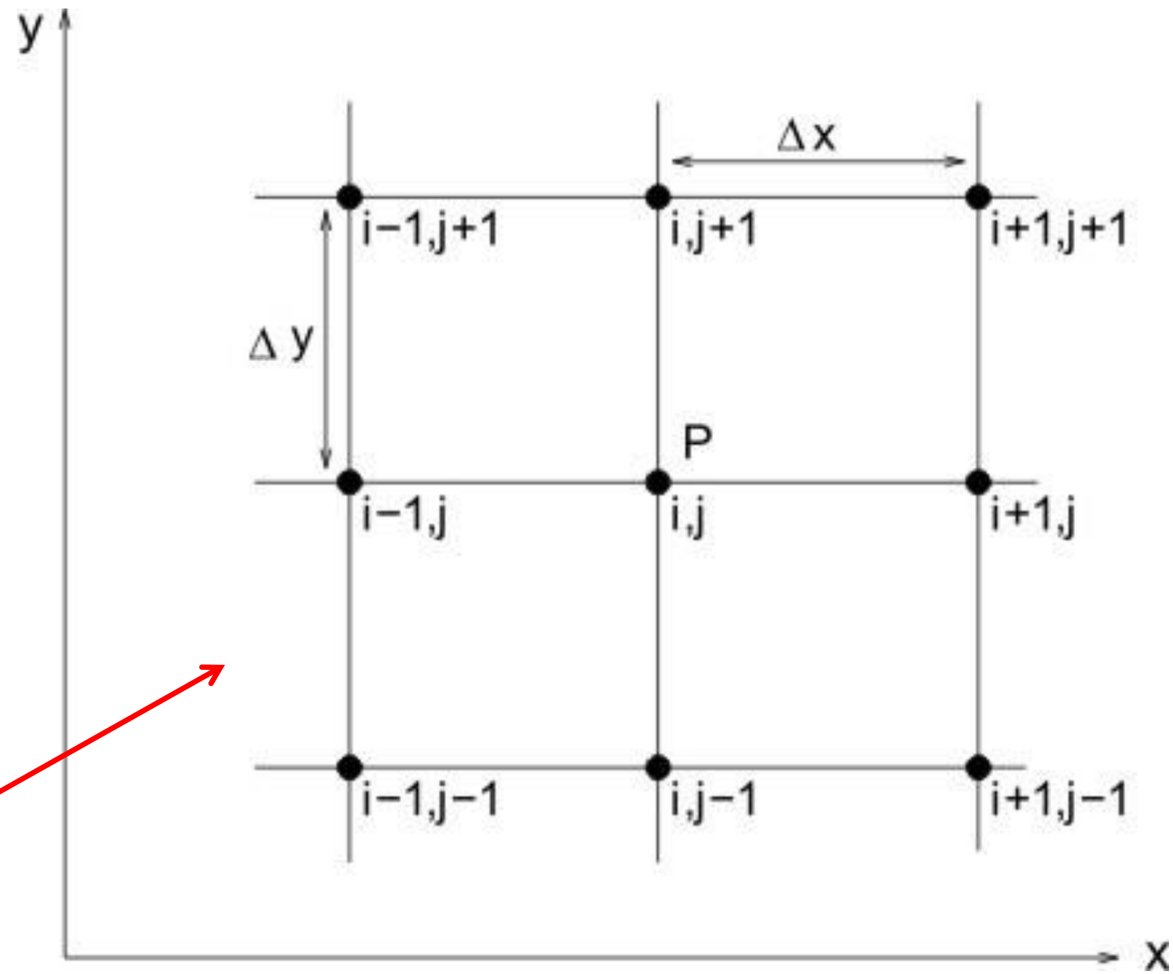
Methods of Discretization

Name of the Method	Process
Finite-Difference Method	The method includes the assumption that the variation of the unknown to be computed is somewhat like a polynomial in x, y, or z so that higher derivatives are unimportant.
Due to limitations of FDM in dealing with problems with increasing physical complexity, new advanced methods were developed. They can be divided in following 2 categories:	
a. **Finite element Method**	It finds solutions at discrete spatial regions (called elements) by assuming that the governing differential equations apply to the continuum within each element.
b. **Spectral Method**	The approximation is based on expansions of independent variables into finite series of smooth functions.
2. **Finite Volume Method**	The calculation domain is divided into a number of non-overlapping control volumes such that there is one control volume surrounding each grid point. The differential equation is integrated over each control volume. Piecewise profiles expressing the variation of the unknown between the grid points are used to evaluate the required integrals.

Basic concept of numerical grid points







Grid points

Classification of Partial Differential Equations

Criteria	Detail	Examples
Order	The order of a PDE is determined by the highest-order partial derivative present in that equation	First order: $\partial\phi/\partial x - G\,\partial\phi/\partial y = 0$ Second order: $\partial^2\phi/\partial x^2 - \phi\,\partial\phi/\partial y = 0$ Third order: $[\partial^3\phi/\partial x^3]^2 + \partial^2\phi/\partial x\partial y + \partial\phi/\partial y = 0$
Linearity	If the coefficients are constants or functions of the independent variables only, then Eq. is *linear. If* the coefficients are functions of the dependent variables and/or any of its derivatives of either lower or same order, then the equation is *nonlinear.*	$a\,\partial^2\phi/\partial x^2 + \mathrm{b}\,\partial^2\phi/\partial x\partial y + c\,\partial^2\phi/\partial y^2 + d = 0$

Elliptic, Parabolic, and Hyperbolic Equations

Linear second-order PDEs in two independent variables are further classified into three canonical forms i.e. **elliptic, parabolic, and hyperbolic**.

The general form of this class of equations is:

$$a\frac{\partial^2\phi}{\partial x^2}+b\frac{\partial^2\phi}{\partial x\partial y}+c\frac{\partial^2\phi}{\partial y^2}+d\frac{\partial\phi}{\partial x}+e\frac{\partial\phi}{\partial y}+f\phi+g=0$$

where coefficients are either constants or functions of the independent variables only.

- The three canonical forms are determined by the following criteria:

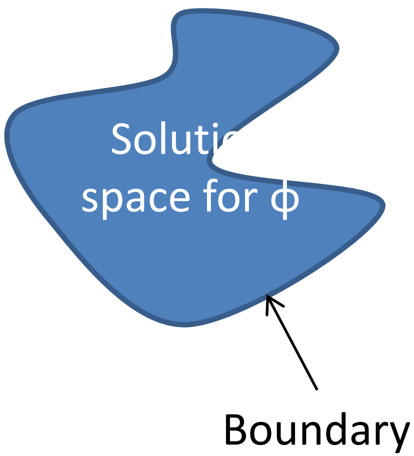
1. *$b^2 - 4ac < 0$ elliptic*
2. *$b^2 - 4ac = 0$ parabolic*
3. *$b^2 - 4ac > 0$ hyperbolic*

Examples of **elliptic, parabolic, and hyperbolic** equations

Elliptic PDE

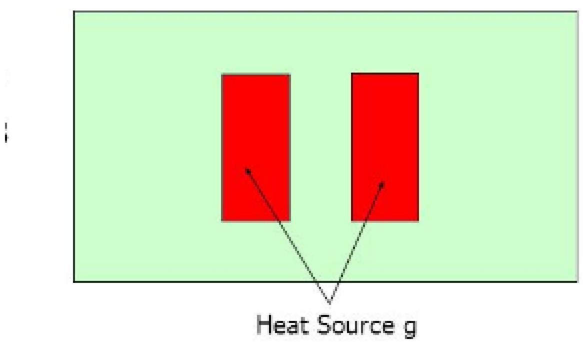
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$ Laplace's equation

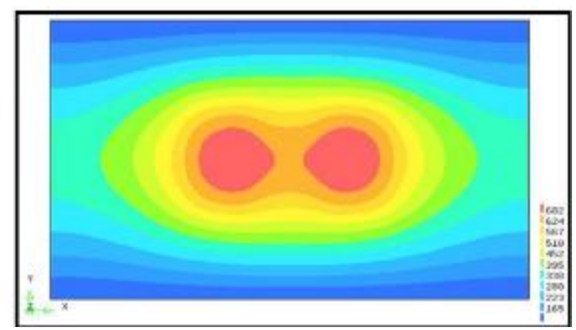
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = f$$ Poisson's equation



This equation requires only boundary condition for the solution.

Temperature profile around computer chips in a printed circuit board





CFD solution

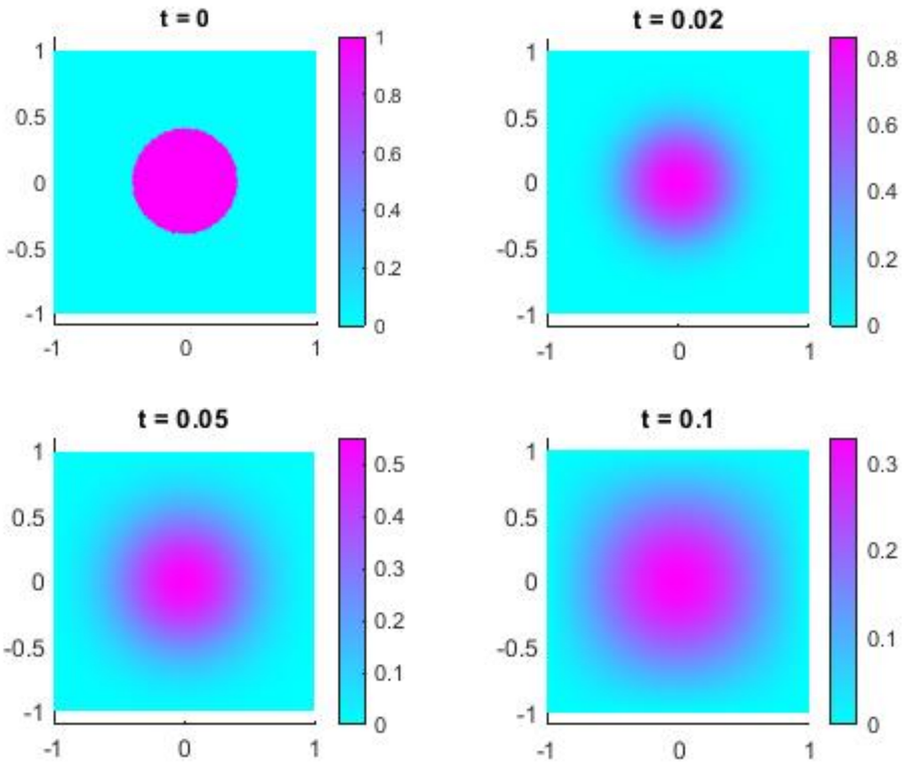
Examples of **elliptic, parabolic, and hyperbolic** equations

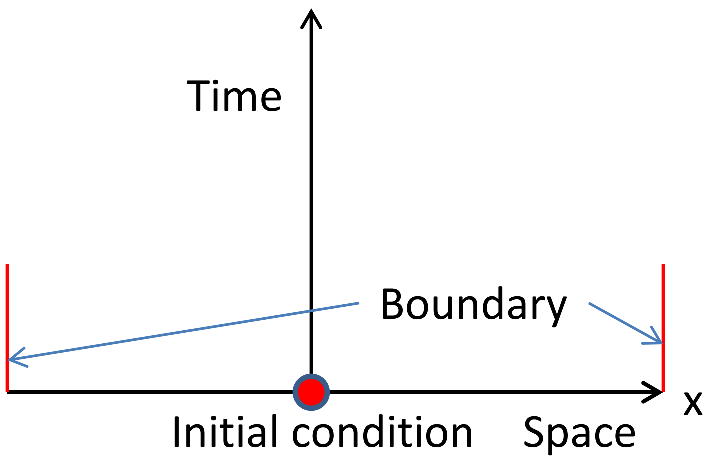
Parabolic PDE

$$\frac{\partial \phi}{\partial t} = \alpha \frac{\partial^2 \phi}{\partial x^2}$$ Transient heat conduction/difusion equation

This equation requires both one initial and two boundary conditions for the solution.

Diffusion of species with time





Time marching problem.

CFD solution