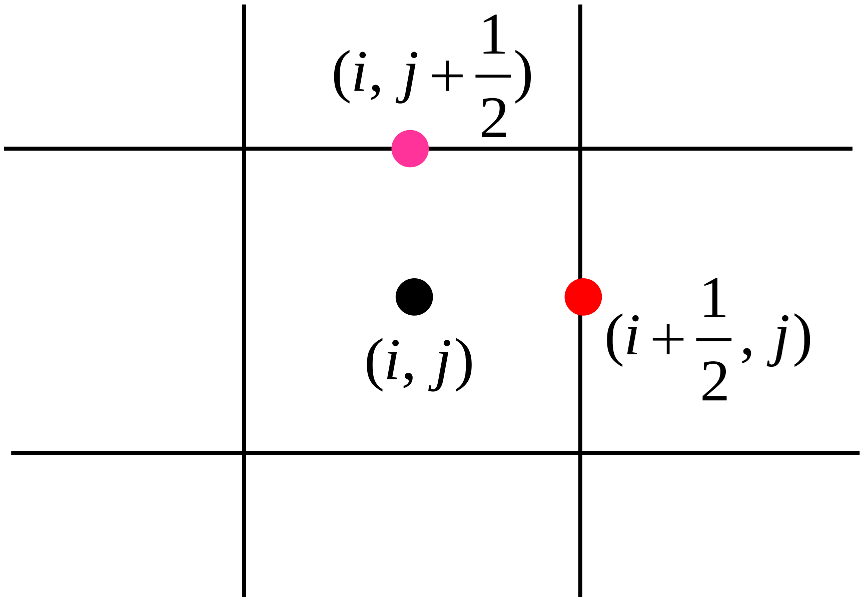
SPECIAL TOPICS CONTD..

Chapter - 15

Discretizatied equations using the SIMPLE Algorithm



Predictor equation

X-momentum equation

$$\frac{u^*_{i+\frac{1}{2},j} - u^p_{i+\frac{1}{2},j}}{\Delta t} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right)^{p+1}_{i+\frac{1}{2},j} = -\frac{1}{\rho}\left(\frac{\partial p^*}{\partial x} \right)^{p+1}_{i+\frac{1}{2},j} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)^{p+1}_{i+\frac{1}{2},j}$$

Now,

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)^{p+1}_{i+\frac{1}{2},j}=\left(\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}\right)^{p+1}_{i+\frac{1}{2},j}=\left(\frac{\partial u^2}{\partial x}\right)^{p+1}_{i+\frac{1}{2},j}+\left(\frac{\partial uv}{\partial y}\right)^{p+1}_{i+\frac{1}{2},j}$$

$$\left(\frac{\partial u^2}{\partial x}\right)^{p+1}_{i+\frac{1}{2},j}=\frac{\left(u^2_{i+\frac{1}{2}+1,j}\right)^{p+1}-\left(u^2_{i+\frac{1}{2}-1,j}\right)^{p+1}}{2\Delta x}=\frac{\left(u^2_{i+\frac{3}{2},j}\right)^{p+1}-\left(u^2_{i-\frac{1}{2},j}\right)^{p+1}}{2\Delta x}$$

$$\left(\frac{\partial uv}{\partial y}\right)^{p+1}_{i+\frac{1}{2},j}=\frac{(uv)^{p+1}_{i+\frac{1}{2},j+1}-(uv)^{p+1}_{i+\frac{1}{2},j-1}}{2\Delta y}=\frac{u^{p+1}_{i+\frac{1}{2},j+1}v^{p+1}_{i+\frac{1}{2},j+1}-u^{p+1}_{i+\frac{1}{2},j-1}v^{p+1}_{i+\frac{1}{2},j-1}}{2\Delta y}$$

Were, (by linear interpolation)

$$v^{p+1}_{i+\frac{1}{2},j+1}=\frac{v^{p+1}_{i+\frac{1}{2},j+\frac{1}{2}}+v^{p+1}_{i+\frac{1}{2},j+\frac{3}{2}}}{2}$$

$$v^{p+1}_{i+\frac{1}{2},j+\frac{1}{2}}=\frac{v^{p+1}_{i,j+\frac{1}{2}}+v^{p+1}_{i+1,j+\frac{1}{2}}}{2}$$

Again,

$$-\frac{1}{\rho}\left(\frac{\partial p^*}{\partial x}\right)_{i+\frac{1}{2},j}^{p+1} = -\frac{1}{\rho}\left(\frac{p^*_{i+1,j} - p^*_{i,j}}{\Delta x}\right)^{p+1}$$

and,

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)_{i+\frac{1}{2},j}^{p+1} = \left(\frac{\partial^2 u}{\partial x^2}\right)_{i+\frac{1}{2},j}^{p+1} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i+\frac{1}{2},j}^{p+1}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i+\frac{1}{2},j}^{p+1} = \frac{u^{p+1}_{i+\frac{1}{2}+1,j} - 2u^{p+1}_{i+\frac{1}{2},j} + u^{p+1}_{i+\frac{1}{2}-1,j}}{(\Delta x)^2} = \frac{u^{p+1}_{i+\frac{3}{2},j} - 2u^{p+1}_{i+\frac{1}{2},j} + u^{p+1}_{i-\frac{1}{2},j}}{(\Delta x)^2}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{i+\frac{1}{2},j}^{p+1} = \frac{u^{p+1}_{i+\frac{1}{2},j+1} - 2u^{p+1}_{i+\frac{1}{2},j} + u^{p+1}_{i+\frac{1}{2},j-1}}{(\Delta y)^2}$$

Similarly for v momentum equation

Pressure correction equation

$$\left(\frac{\partial^2 p'}{\partial x^2}\right)_{i,j}^{p+1} + \left(\frac{\partial^2 p'}{\partial y^2}\right)_{i,j}^{p+1} = \frac{\rho}{\Delta t}\left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y}\right)_{i,j}$$

$$\Rightarrow \left(\frac{p'_{i+1,j} - 2p'_{i,j} + p'_{i-1,j}}{(\Delta x)^2}\right)^{p+1} + \left(\frac{p'_{i,j+1} - 2p'_{i,j} + p'_{i,j-1}}{(\Delta y)^2}\right)^{p+1}$$

$$= \frac{\rho}{\Delta t}\left(\frac{u^*_{i+\frac{1}{2},j} - u^*_{i-\frac{1}{2},j}}{\Delta x}\right) + \frac{\rho}{\Delta t}\left(\frac{v^*_{i,j+\frac{1}{2}} - v^*_{i,j-\frac{1}{2}}}{\Delta y}\right)$$

Velocity correction equation

$$u^{p+1} = u^* - \frac{\Delta t}{\rho}\left(\frac{\partial p'}{\partial x}\right)^{p+1}$$

$$\Rightarrow u^{p+1}_{i+\frac{1}{2},j} = u^*_{i+\frac{1}{2},j} - \frac{\Delta t}{\rho}\left(\frac{p'_{i+1,j} - p'_{i,j}}{\Delta x}\right)^{p+1}$$

Boundary conditions:

At walls

$$u = 0,\ u^* = 0$$

$$v = 0, v^* = 0$$

No-slip

$$\frac{\partial p}{\partial n} = 0, \frac{\partial p'}{\partial n} = 0$$

At outlet

$$\frac{\partial u}{\partial n} = 0$$

$$\frac{\partial v}{\partial n} = 0$$

Fully-developed

$$p = specified \Rightarrow p' = 0$$

(*n* is the surface normal)