Ref. No.: **Ex/IEBE/Math/T/113/2019(OLD)**

BACHELOR OF INST. & ELECTRONICS ENGINEERING EXAMINATION – 2019 (OLD)

(1^{ST} YEAR 1^{ST} SEMESTER)

MATHEMATICS – IJ

FULL MARKS : 100 TIME : 3 HOURS

ANSWER ANY 10 QUESTIONS:

1. (a) Find $y = x^{2n}$, where n is a positive integer. Show that

$$y_n = 2^n \{1.3.5\ldots(2n-1)\}x^n.$$

(b) If $y = log\left(x + \sqrt{x^2+1}\right)$, then show that

$$(x^2+1)y_{n+2} - (2n+1)xy_{n+1} + n^2 y_n = 0.$$ 4+6

2. (a) In the Mean Value Theorem $f(h) = f(0) + hf'(\theta h), 0 < \theta < 1$, if $f(x) = \frac{1}{1+x}$ and $h = 3$, then find the value of θ.

(b) State Rolle's theorem. Verify Rolle's theorem for the following function $f(x) = |x-1|$ in [0,2]. 5+5

3. (a) Show that $\log(1+x) > x - \frac{1}{2}x^2$, if $x > 0$.

(b) Expand the function $\sin^3 x$ in power of x in a finite form with Lagrange's form of remainder. 5+5

4. (a) Expand cosax in an infinite series in powers of x.

(b) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. 5+5

5. (a) Show that $x^2 \log(1/x)$ is a maximum for $x = 1/\sqrt{e}$.

(b) If $u = e^{xyz}$, then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$. 5+5

6. (a) If $u = cos^{-1}\{(x+y)/(\sqrt{x}+\sqrt{y})\}$, show that

$$x\frac{\partial u}{dx} + y\frac{\partial u}{dy} + \frac{1}{2}cotu = o.$$

(b) Find the maximum and minimum values of $x^2y^2 - 5x^2 - 8xy - 5y^2$. 5+5

7. (a) If $u = f(y-z, z-x, x-y)$, prove that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$$

(b) If u be a homogeneous function of x and y of degree n prove that

$$\left(x\frac{\partial}{dx}+y\frac{\partial}{dy}\right)^2 u = n(n-1)u.$$ 5+5

8. (a) If $\cos\alpha + cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then show that

$$\cos 3\alpha + cos3\beta + cos3\gamma = 3\cos(\alpha + \beta + \gamma)$$

(b) Expand sin 7θ in powers of sin θ. 5+5

9. (a) If $\tan(\alpha + i\beta) = x + iy$, show that

$$x^2 + y^2 + 2x \cot 2\alpha = 1.$$

(b)Find the general value of i^i. 5+5

10. (a) Show that $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} = 2$

(b) Evaluate $\int_0^{\pi/4} \log(1 + \tan x)\, dx$ 5+5

11. (a) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta$

(b) Show that $\int_0^1 x^m(1-x)^p\, dx = \frac{1}{n}B\left(\frac{m+1}{n}, p+1\right)$. 5+5

12. (a) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\, dx.$

(b) Show that $\beta(m+1, n) = \frac{m}{m+n}\beta(m, n)$. 5+5

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