10. A tightly stretched string with fixed end points x = 0 and x = L is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from rest from this position, find the displacement y(x,t). 10

11. Given that $f(x) = x + x^2$ for $-\pi < x < \pi$ and $f(x) = \pi^2$ for $x = \pm\pi$. Expand f(x) in Fourier series, and show that

$$x + x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left\{\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right\}$$ 10

12. Obtain the half range cosine and sine series expansion for f (x) = x in the interval $0 \le x \le \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$ 10

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BACHELOR OF (I.E.E.) ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - III J

Time : Three hours Full Marks : 100

Notations/Symbols have their usual meanings.

Answer any ***ten*** questions.

1. (a) Show that the series $1 + r + r^2 + r^3 + \dots \alpha$

   (i) converges if $|r| < 1$, (ii) diverges if $r \ge 1$, and (iii) oscillates if $r \le -1$

   (b) Test for convergence the following series :

   $$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots\dots \alpha$$ 6+4

2. Discuss the convergence of the following series :

   (a) $\sum_{n=1}^{\alpha} \frac{n!}{(n^n)^2}$

   (b) $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \frac{5!}{5^5} + \dots\dots\alpha$

3. (a) Solve : $\frac{dy}{dx} = (4x + y + 1)^2$, given that y = 1 when x = 0.

(b) Solve : $y'' + 4y' + 4y = 3 \sin x + 4 \cos x$, given that $y = 1$ and $y' = 0$ when $x = 0$. 4+6

4. (a) Solve : $(D^3 + D^2 + 4D + 4)y = 0$

(b) Using the method of variation of parameters, solve the following differential equation :

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$ 3+7

5. (a) Show that the following equation is exact and hence solve it :

$$\left\{ y\left(1 + \frac{1}{x}\right) + \cos y \right\} dx + (x + \log x - x \sin y)\, dy = 0$$

(b) Solve : $(D^3 + 1)y = \cos(2x - 1)$ 5+5

6. (a) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

(b) Find the series solution of

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$ 4+6

7. (a) Form a PDE by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$.

(b) Form a PDE by eliminating the arbitrary function from

(i) $f(x^2 + y^2, z - xy) = 0$

(ii) $z = f\left(\frac{xy}{z}\right)$ 2+(5+3)

8. (a) Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2 \sin y$ when x = 0, and z = 0 when y is an odd multiple of $\frac{\pi}{2}$.

(b) Solve the following PDEs :

(i) $\frac{y^2 z}{x} p + xzq = y^2$

(ii) $y^2 p - xyq = x(z - 2y)$ 4+(3+3)

9. Obtain the various possible solutions of the one-dimensional heat conduction equation $\frac{\partial u}{\partial t} = |\alpha \frac{\partial^2 u}{\partial x^2}$, by the method of separation of variables. Identify the solution which is appropriate with the physical nature of the equation. Justify your answer. 10