Ref. No.: Ex/IEBE/MATH/T/114/2019(Old)

Bachelor of Engineering
Instrumentation and Electronics Engineering
Examination, 2019 (Old)
(1st Year, 1st Semester)
Mathematics-IIJ

Time : Three hours **Full Marks:100**

(Notations and symbols have their usual meanings.)

Answer Question no 1 and any eight from the rest.

1. Write down the Serret-Frenet formula . 4

2. a) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & a^3 \end{vmatrix} = \text{(a-b)(b-c)(c-a)(a+b+c)}$$

(b) Solve the system of equations by Cramer's rule.

$$\text{x+ y+ z} = 1$$
$$\text{ax+ by+ cz} = 1$$
$$\text{a}^2x + b^2y + c^2z = 1$$

where, $a \neq b \neq c$. $6 + 6$

3. (a) If $(I_n + A)^{-1}(I_n - A)$ is a real skew symmetric matrix, prove that the matrix A is orthogonal.

(b) Find the matrix A if

$$\text{adj A}= \begin{pmatrix} 1 & 3 & -4 \\ -2 & 2 & -2 \\ 1 & -3 & 4 \end{pmatrix}.$$

$7 + 5$

4. (a) Find the rank of the following matrix

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

(b) Determine the values of a, b for which the system of equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + az &= b \end{aligned}$$

has (i)only one solution, (ii) no solution (iii) many solutions.

5 + 7

5. (a) Find the eigen values and the corresponding eigen vectors of the following matrix

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

Hence, find A^{-1}.

6+6

6. (a) If $2\cos\theta = x + \frac{1}{x}$ and θ is real, prove that $2\ cosn\theta = x^n + \frac{1}{x^n}$, n being an integer.

(b) For two complex numbers z_1, z_2, if $|z_1| = |z_2| = 1$ and, $arg\ \ z_1 + arg\ \ z_2 = 0$, then show that $z_1 z_2 = 1$.

6 + 6

7. (a) Show that four points whose position vectors are (4, 8, 12), (2, 4, 6), (3, 5, 4) and (5, 8, 5) are coplanar.

(b) Determine λ, so that $(\lambda\hat{i} - 4\hat{j} + 3\hat{k})$ and $(3\hat{i} + \lambda\hat{j} - 2\hat{k})$ are perpendicular.

(c) Prove by vector method that the medians of a triangle are concurrent.

5 + 3 + 4

8. (a) Show that $\bar{a}.\bar{b} = 0$, iff $|\bar{a} + \bar{b}| = |\bar{a} - \bar{b}|$.

(b) Show that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$ iff $(\bar{c} \times \bar{a}) \times \bar{b} = 0$.

(c) Show that $[\bar{a} + \bar{b}\ \ \bar{b} + \bar{c}\ \ \bar{c} + \bar{a}] = 2[\bar{a}\ \ \bar{b}\ \ \bar{c}]$.

4 + 4 + 4

9. (a) Find the curvature and torsion of the twisted cubic $\bar{r} = (2t, t^2, \frac{t^3}{3})$ at $t = 1$. Also find the equation of the osculating plane.

(b) Find the equation of the sphere having (2,1,2) and (1,0,1) as the ends of a diameter.

8+4

10. (a) Obtain the equation of the right circular cylinder whose axis is given by $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z}{2}$ and the radius of circular region is 5 unit.

(b) Find the curvature and torsion of the curve
$\bar{r} = (tan^{-1}s)\hat{i} + \frac{1}{2}\sqrt{2}\ \ log(s^2+1)\hat{j} + (s - tan^{-1}s)\hat{k}$

5 + 7

11. (a) Find the shortest distance between two skew lines $\bar{r} = \bar{r_1} + t\bar{\alpha}$ and $\bar{r} = \bar{r_2} + t\bar{\beta}$ where t is a scalar and $\bar{r_1}, \bar{r_2}, \bar{\alpha}, \bar{\beta}$ are vectors with co-ordinates (1, -2, 3), (2, 1, 1), (-2, 2, -1) and (-3, 1, 2) respectively.

(b) If $\bar{r} = a\ cost\hat{i} + a\ sint\hat{j} + at\ tan\alpha\hat{k}$, then prove that
$\left[\frac{d\bar{r}}{dt}\ \ \frac{d^2\bar{r}}{dt^2}\ \ \frac{d^3\bar{r}}{dt^3}\right] = a^3 tan\alpha.$

6 + 6