EX/ IT/T/326B/2019

B. INFO. TECH 3RD YEAR 2ND SEMESTER EXAMINATION, 2019

MATHEMATICAL METHODS

Time: Three hours   Full Marks: 100

Different parts of the same question should be answered together.

CO1 [20]

[1] Answer the following questions.

(a) The Stanley Cup winner is determined in the final series between two teams. The first team to win 4 games wins the Cup. Suppose that Dallas Stars advance to the final series, and they have a probability of 0.55 to win each game, and the game results are independent of each other.
Suppose that the series continues until Dallas Stars win 4 games, even if the other rival wins the Cup earlier.
i) What is the expected number of games to be played?
ii) Find the probability that the series consists of less than 10 games.
iii) Find the probability that by the end of the series, the other rival wins at least 3 games.

[10]

(b) Suppose that the number of inquiries arriving at a certain interactive system follows a Poisson distribution with arrival rate of 12 inquiries per minute.
Find the probability of 10 inquiries arriving
i) in a 1-minute interval;
ii) in a 3-minute interval.
iii) What is the expectation and the variance of the number of arrivals during each of these intervals?

[10]

CO2 [20]

[2] *Answer* either (a) or (b) *in this block*

(a) Suppose that a computing system consists of (i) a file server, (ii) two workstations and (iii) computing network connecting them. The system operational as long as one of the Workstations and the file-server are operational. Computer network is assumed to be fault free. Find out
(i) The system reliability.
(ii) The system mean time to failure
(iii) The system instantaneous availability
(iv) The system steady state availability

[20]

(b) Consider a series connection of two components, with respective lifetimes X and Y. The joint pdf of the lifetimes is given by

$$f(x,y) = \begin{cases} \frac{1}{200}, & (x,y) \in A \\ 0, & elsewhere \end{cases}$$

where A is the triangular region in the (x, y) plane with the vertices (100, 100), (100, 120), and (120, 120). Find the reliability expression for the entire system.

[20]

CO3 [20]

[3]

(a) Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates $\lambda_1=1$and $\lambda_2=2$, respectively. Find the probability that the second arrival in $N_1(t)$ occurs before the third arrival in $N_2(t)$. *Hint:* One way to solve this problem is to think of $N_1(t)$ and $N_2(t)$ as two processes obtained from splitting a Poisson process.

[10]

(b) Let $\{N(t), t\in[0,\infty)\}$ be a Poisson process with rate λ. Find the probability that there are two arrivals in (0,2] and three arrivals in (1,4].

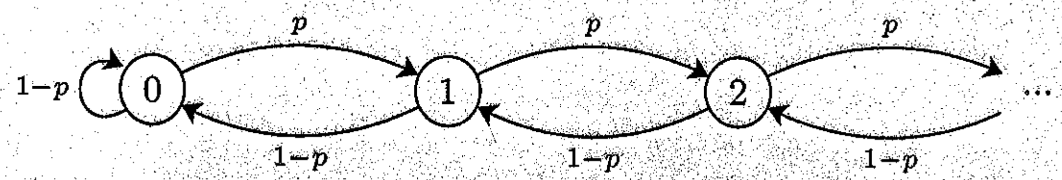
[10]

CO4 [20]

[4]

(a) Consider the Markov chain shown in Figure. Assume that $1/2<p<1$. Does this chain have a limiting distribution? For all $i,j\in\{0,1,2,\cdots\}$ $i,j\in\{0,1,2,\cdots\}$, find

$$\lim_{n\to\infty}P(X_n=j|X_0=i).$$



[10]

(b) Consider the Markov chain with three states, $S=\{1,2,3\}$ that has the following transition matrix.

$$P=\begin{matrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{matrix}$$

(i) Draw the state transition diagram for this chain.

(ii) If we know $P(X_1=1)=P(X_1=2)=1/4$, find $P(X_1=3,X_2=2,X_3=1)$

[10]

CO5 [20]

[5] *Answer any* two out of (a), (b) and (c) from this block:

(a) A small internet café has two computer terminals. The arrival rate of internet users in the café is 10 users per hour. Each user spends 10 minutes on the computer. The arrival and service process follow exponential distribution.

(i) What is the probability that both computers are free?

(ii) What is the probability that a customer can use the computer after arriving?

(iii) What is the probability that a customer will find no queue on arrival?

(iv) Find the average number of customers in the system?

[10]

(b) Consider the following situation.

- Customers arrive according to a Poisson process with rate λ.
- The system has a finite capacity of c customers including the one in service.
- There is only one server.
- Service times are exponential with rate μ.

(i) Derive the probability of having no customers.

(ii) Derive the probability of having N customers.

(iii) Derive the average number of customers in the system,

[10]

(c) Consider an *M/M/*1 queuing system in which the total number of jobs is limited to *n* owing to a limitation on queue size.

(i) Find the steady state probability that an arriving request is rejected because the queue is full.

(ii) Find the steady-state probability that the processor is idle.

(iii) Find the throughput of the system in the steady state.

(iv) Given that a request has been accepted, find its average response time.

[10]