Ex/IT/T/313/2019

Bachelor of Information Technology Examination, 2019
(3^{rd} year, 1^{st} semester)
Graph Theory

Times : Three hours Full Marks : 100

Use Separate Answer scripts for each Part.

Part-I

CO1 1 **Attempt any three (3)** 5x3=15

a. Find all non-isomorphic simple graphs with four vertices.
b. A regular graph with degree 3 is called cubic. Prove that every cubic graph has even number of vertices.
c. Define complement graph. Find the complement of $K_{4,3}$.
d. In a connected graph any two longest paths have a vertex in common.

CO2 2 **Attempt any two (2)** 5x2=10

a. Prove that every connected graph has a spanning tree.
b. Construct the tree corresponding to Prüffer's code 1,2,2,7,6,6,5.
c. A tree with order n (≥2), prove that number of pendant vertices is at least 2.

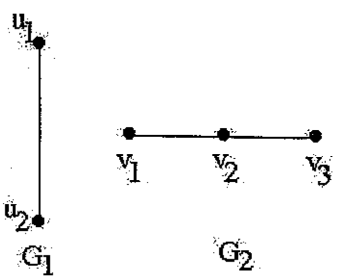
CO3 3 **Attempt any two (2)** 5x2=10

a. Let G(V,E) be a product of two graphs G_1 and G_2 i.e., $G= G_1 \times G_2$, where $V=\{(v_1, v_2) \mid v_1 \in V_1$ and $v_2 \in V_2\}$ and $E=\{ (u, v) \mid u=(u_1, u_2), v=(v_1, v_2)$ and either $u_1=v_1$ and $(u_2, v_2)\in E_2$ or $u_2=v_2$ and $(u_1, v_1) \in E_1\}$. Consider following two graphs and compute the product of these two.
b. If a graph G is a tree, prove that every edge of G is a cut set.
c. A vertex *v* of G is a cut vertex if and only if there are vertices *u* and w adjacent to *v* such that *v* is on every *u-w* path.

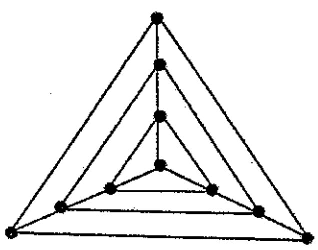
CO4 4 Find a maximum independent set, a minimum vertex cover, a minimum dominating set and chromatic number of the graph given below. 7

5 **Attempt any two (2)** 4x2=8

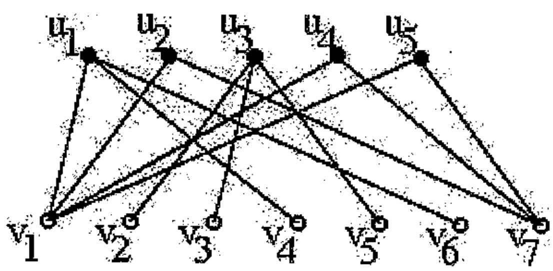
a. G is a planar connected graph with 'n' vertices, 'e' edges and 'f' faces. Starting from a spanning tree of G prove that f= e-n+2.
b. Draw a maximum planar graph which is not hot Hamiltonian.
c. Find the chromatic number of the given graph.



Graphs for question 3.a



Graph for question 4



Graph for question 5.c

CO1: Describe the representation of graph and illustrate and analyze different characteristics and properties different types of graphs (K3)

CO2: Describe different types of trees such as (i) rooted tree (ii) spanning tree etc, and illustrate and analyze their properties. (K3)

CO3: Apply operations like Union, Deletion, and decomposition of graphs and illustrate Cut vertex and cut edge and analyze their properties. (K3)

CO4: Illustrate Planer graph and their properties, Graph Coloring and Matching. (K4)

EX/IT/T/313/2019

B.E. INFORMATION TECHNOLOGY THIRD YEAR FIRST SEMESTER – 2019

(3rd Year; 1st Semester)

Subject: Graph Theory (Part II)

Time: Three hours

Full Marks: 50

CO5 [30]	a) How many n digit binary words can be possible, where the first and last bit will be 0? (5) b) Suppose that in a group of 6 persons each pair is either friends or enemies. Prove that among 6 either there are 3 persons who are mutual friends each is a friend of the other or else there are 3 persons who are mutual enemies each is an enemy of the others. Also show that this is not true for 5 persons. (7+7) c) How many of the numbers 1 through 200 are even or a multiple of 6? (6) d) Show that if there are 30 students in a class, then at least two have last names that begin with the same letter. (5) OR a) A multiple-choice test contains 10 questions. There are four possible answers for each question. i) In how many ways can a student answer the questions on the test if the student answers every question? ii) In how many ways can a student answer the questions on the test if the student can leave answers blank? (4+4) b) Show that (k!)! is divisible by (k!)^(k-1)! (10) c) How many ways can n homework assignments be returned to n students such that no student gets her own homework back? (12)
CO6 [20]	a) Find the number of integer solutions to, x1+x2+x3+x4=21 (10) b) (i) Find a recurrence relation for a(n), the number of ways of arranging n distinct objects in a row. (ii) Solve: (b) a(n) = a(n-2), a(0) = a(1) = 1 (5+5) OR a) Write a recursive function for finding the number of n digit numbers such that no number consists of two consecutive 1s. Construct the Basis of the function and solve it. (4+1+5) b) Find the generating function for a(r), the number of ways to select r balls from a pile of five green, three white, five blue, and six gold balls. (5)

CO5: Apply and Evaluate basic counting rules, pigeon-hole principle, principle of inclusion-exclusion (K3).

CO6: Apply and Evaluate Generating Function and Recurrence Relations. (K3)