BACHELOR OF INFORMATION TECHNOLOGY ENGG.
EXAMINATION - 2019
(2^{ND} YR. 2^{ND} SEM.)

MATHEMATICS-IV(MODULES 8 & 12)

Time: Three hours   Full Marks: 100

Group-A (Probability & Statistics)

CO-1(Total Marks: 10)

Answer any one question:

1. N letter to each of which corresponds an envelope, are placed in the envelopes at random. What is the probability that (i) no letter is placed in the right envelope? (ii) exactly r letters are placed in the right envelopes? 10
2. A player tosses a coin and is to score one point for every head turned up and two points for every tail. He is to play on until his score reaches or passes n. If P_n is the chance for attaining exactly n, show that $P_n = \frac{1}{2}(P_{n-1} + P_{n-2})$ and hence find the value of P_n. 10

CO-2(Total Marks: 20)

Answer any two questions:

3. The joint probability density function of the random variable X, Y is:
$$f(x,y) = k(3x+y), when\ 1 \le x \le 3, 0 \le y \le 2$$
$$0, elsewhere$$
Find: (i)The Value of k; (ii)P(X+Y < 2) (iii)The marginal distribution of X & Y
Investigate whether X & Y are independent. 10
4. If the probability density function of a random variable X is given by:
$$f(x) = ce^{-(x^2+2x+3)}, -\infty < x < \infty$$
Find the value of c, the expectation& variance of the distribution. 10
5. For a bivariate continuous distributions the density function is $f(x,y) = 3x^2 - 8xy + 6y^2, 0 < x < 1, 0 < y < 1$, find the conditional expectation of Xgiven $Y = y$ and hence write down the regression curve for Xon Y. 10

CO-3(Total Marks: 20)

Answer any two questions:

6. (i) A random variable X has a density function f(x) given by:
$$f(x) = e^{-x} for\ x \ge 0$$
$$0, elsewhere$$
Show that Tchebycheff's inequality gives $P\left(|X - 1| \ge 2\right) \le \frac{1}{4}$

And Show that actual probability is e^{-3} .

(ii)A random variable X has probability density function $12x^2(1-x)$, $(0 < x < 1)$, $compute$ $P(|X-m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality. 5+5

7. The joint probability density function of the random variable x and y is: 10

$$f(x,y) = k(1-x-y) \; for \; x \geq 0, y \geq 0, x+y \leq 1$$
$$0, \quad elsewhere$$

Where k is constant. Find: (i)The mean value of y when $x = \frac{1}{2}$ (ii)The covariance of x and y.

8. From the following data, obtain the line of regression of X on Y 10

sales$X$	91	97	108	121	67	124	51	73	111	53
purchases$Y$	71	75	69	97	70	91	39	61	80	47

Group-B (Discrete Mathematics)

CO-4(Total Marks: 20)

9. If $f, g, h: B^n \to B$, then prove that(using table):

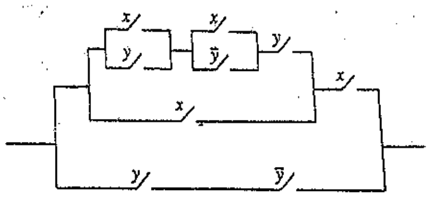
(i)$fg + \overline{f}h + gh = fg + \overline{f}h$ (ii)$fg + f\,\overline{g} + \overline{f}g + \overline{f}\overline{g} = 1$ 5+5

10. (a)Convert c.n.f. to d.n.f. of:

$$(x+\overline{y}+z)(x+y+\overline{z})(x+\overline{y}+\overline{z})(\overline{x}+y+z)(\overline{x}+y+\overline{z})(\overline{x}+\overline{y}+z)$$

(b)Find d.n.f. and c.n.f. of: $f(x,y,z) = \overline{\overline{x+y} + (\overline{x}z)}$ 5+5

11. Find the Boolean Expression of the following switching circuit and obtain its equivalent switching circuit and show by a table that two circuits are equivalent. 10



Group-C (Real Analysis)

CO-5(Total Marks: 20)

Answer any two questions:

12. If n be a positive integer ≥ 2 and a be a positive real number, show that there exists a unique positive real number x such that $x^n = a$. 10

13. Let S be a subset of Q defined by $S = \{x \in Q: x > 0 \; \& \; x^2 < 2\}$. Show that S is a none-empty subset of Q bounded above but $sup\ S$ does not belong to Q. 10

14. (i)Prove that for each $n \in N$; $1 + 2 + 3 + \cdots \ldots\ldots + n = \frac{n(n+1)}{2}$ $\forall n \in N$

(ii)Prove that for all $n \in N$, $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is an even integer.

CO-6(Total Marks: 10)

Answer any one question:

15. Let $S = \{(-1)^m + \frac{1}{n} : m \in N, n \in N\}$

(i)Show that 1 and -1 are limit points of S. (ii)Find S' (The Derived set of S). 5+5

16. (i)Using the definition of a compact set, prove that a finite subset of R is a compact set in R. 5

(ii)Let K be a compact subset of R & $F \subset K$be a closed subset in R. Prove that F is compact in R. 5

COS:

CO -1:Define and illustrate (i)the concept of probability,(ii)measures of central tendency, dispersion, correlation and (iii)random variables, P.D.F.,C.D.F.(K2,A2)

CO-2: Solve problems using the concept of probability and the statistics like measures of central tendency, dispersion and correlation from the given data. (K3, A2)

CO-3: Solve problems using random variables, P.D.F., C.D.F. (K3, A2)

CO-4:Define illustrate and apply the basic principles and theorems of mathematical logic, Boolean Algebra and switching function. (K3, A2)

CO-5:Comprehenced and illustrate different properties of the sets of real numbers like Archimedean property. (K2, A2)

CO-6:Discuss and illustrate limiting point, continuity and differentiability of the real valued function. (K2, A2)