BACHELOR OF INFORMATION TECHNOLOGY ENGG. EXAMINATION - 2019

(2ND YR. 1ST SEM.)

MATHEMATICS-III(Modules 3 & 10)

Time: Three hours — Full Marks: 100

Answer any **Ten** questions — 10×10

1. (i) Expresss $A = \begin{pmatrix} 1 & 3 & 4 \\ 7 & 2 & 6 \\ 2 & 8 & 1 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

   (ii) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A - 5I_3 = 0$, Hence obtain a matrix B such that $AB = I_3$. 5+5

2. (i) Find the value of $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -b & a & b \\ -d & c & -b & a \end{vmatrix}$, using Laplace's method.

   (ii)Solve by Cramer's rule $x - y + z = 0; 2x + 3y - 5z = 7; 3x - 4y - 2z = -1$. 5+5

3. (a)Obtain the fully reduced normal form of the matrix $A = \begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$ and find its rank.

   (b)Show that the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix}$ is non-singular and express it as a product of elementary matrices. 6+4

4. Statement of Cayley Hamilton theorem. Verify Cayley Hamilton theorem for the matrix A. Express A^{-1} as a polynomial in A and then compute A^{-1}, where $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$. 10

5. Diagonalise the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. 10

6. Prove that the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represent a pair of planes, if $bc + 2fgh - af^2 - bg^2 - ch^2 = 0$, and also prove that the angle between the planes is $\tan^{-1} \frac{2(f^2+g^2+h^2-bc-ca-ab)^{\frac{1}{2}}}{a+b+c}$. 10

7. (a) Show that the locus of a variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$.

   (b) A variable plane at a constant distance P from the origin meets the axes at A,B,C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$. 5+5

8. (a)Show that the straight lines whose d.cs. are given by $al + bm + cn = 0, fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ *and parallel if* $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$.

   (b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0, x - 2y + 3z + 1 = 0$ is a great circle. 5+5

9. Show that the equation of the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$ and if $2d$ be the shortest distance, then find the value of $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$=? 10

10. Prove that the set $G = \{(\cos\theta + i\sin\theta): \theta \text{ runs over rational number}\}$ forms an infinite abelian group with respect to ordinary multiplication. 10

11. Write down the axioms for a ring and for a field. Show that the set of all real numbers of the form $a + b\sqrt{2}$, a and b are rational numbers, is a field under usual addition and multiplication. 10

12. Show that all the roots of the equation $x^6 = 1$ from a cyclic group of order 6, under usual multiplication of complex numbers. 10

13. If $(R, +, .)$be a ring such that $a^2 = a$, for all $a \in R$, then prove that

    (i)$a + a = 0$ *for all* $a \in R$

    (ii)$a + b = 0 \Rightarrow a = b$ *for all* $a, b \in R$.

    (iii)$a.b = b.a$ *for all* $a, b \in R$ 10