EX/ IT/MATH/T/114/2019(OLD)

BACHELOR OF INFORMATION TECHNOLOGY ENGG. EXAMINATION - 2019
(1^{ST} YR. 1^{ST} SEM.)
MATHEMATICS-I-(MODULE I & II)OLD

Time: Three hours | Full Marks: 100

Answer any **Ten** questions 10×10

1. State Sandwich theorem. Prove that the sequence $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots\ldots+\frac{1}{\sqrt{n^2+n}}\right)=1$, using Sandwich theorem. 10

2. (a)Test the convergence of the series $\sum \frac{4.7\ldots\ldots\ldots(3n+1)}{1.2\ldots\ldots n} x^n$

   (b)Test the series for convergence, if $\beta-\alpha \neq 1$

   $1+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\frac{(1+\alpha)(2+\alpha)(3+\alpha)}{(1+\beta)(2+\beta)(3+\beta)}+\cdots\ldots\ldots\ldots\ldots\ldots$ 5+5

3. Prove that the infinite series $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots\ldots\ldots.+\frac{1}{n^p}+\cdots\ldots\ldots$ converges if $p>1$, diverges if $p\leq 1$. 10

4. (a) Find the value of p and q in order that $\lim_{x\to 0}\frac{x(1-p\cos x)+q\sin x}{x^3}=\frac{1}{3}$

   (b) Use MVT, show that $\frac{\tan x}{x}>\frac{x}{\sin x}$ when $0<x<\frac{\pi}{2}$. 5+5

5. If $y=e^{\tan^{-1}x}=a_0+a_1x+a_2x^2+a_3x^3+\ldots\ldots\ldots\ldots$,find the value of (i) $(1+x^2)y_2+(2x-1)y_1$; (ii) $(1+x^2)y_{n+2}+\{2(n+1)x-1\}y_{n+1}+n(n+1)y_n$ and (iii) $(n+2)a_{n+2}-a_{n+1}+na_n$ 10

6. If $u=\log(x^3+y^3+z^3-3xyz)$, then find the value of (i) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$ (ii) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u$ and (iii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}$ 10

7. (a)Given the function $f(x,y)=\frac{xy(x^2-y^2)}{x^2+y^2}$, $(x,y)\neq(0,0)$

   0, $(x,y)=(0,0)$

   Find from definition, $f_{xy}(0,0)=?$ *and* $f_{yx}(0,0)=?$ verify whether $f_{xy}(0,0)=f_{yx}(0,0)$. 10

8. If $u=\sin^{-1}\left\{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$ show that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right)$ 10

9. (a) Prove that $\Gamma m\Gamma(1-m)=\pi\, cosec\, m\pi, 0<m<1$ 10

10. (a) Assuming the integral to be convergent, show that $\int_0^{\frac{\pi}{2}}\log\sin x\ dx=\frac{\pi}{2}\log\frac{1}{2}$.

    (b)Applying Beta and Gamma function, find the value of $\int_0^1 x^4\ (1-x^2)^3\,dx$. 6+4

11. Calculate by Simpson's one –third rule, the value of the integral $\int_0^1\frac{x}{1+x}\,dx$ correct upto three significant figures, by taking six intervals, also find its error. 10