Ex:/PRN/MATH/T/221/2019

BACHELOR OF PRINTING ENGINEERING EXAMINATION, 2019

(2nd Year, 2nd Semester)

Mathematics - IV R

Time : Three hours Full Marks : 100

Symbols/Notations have there usual meanings.

Answer any ***five*** questions.

1. (a) Obtain Fourier series for $f(t) = 1 - t^2, -1 < t < 1$.

   (b) Obtain Fourier series for

$$f(x) = \begin{cases} -x+1, & for -\pi \le x \le 0 \\ x+1, & for\, 0 < x \le \pi \end{cases}$$

   Hence show that $\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{\pi^2}{8}$. 10+10

2. (a)(i)Prove that a proper vector $\vec{U}$ has a constant length is that $\vec{U}.\frac{d}{dt}\vec{U} = 0$.

   (ii) Prove that a necessary and sufficient condition that a proper vector $\vec{U}$ always remain parallel to a fixed line is $\vec{U} \times \frac{d}{dt}\vec{U} = 0$.

(Turn Over)

(b) Evaluate $\frac{d}{dt}\left[\vec{r}\ \vec{r}^{/}\ \vec{r}^{//}\right]$

(c) Calculate $\int_c \overrightarrow{F.dr}$ where

(i)C is a triangle in x-y plane with vertices (0,0), (2,0), (2,1)

(ii) C is a curve $x^2 + y^2 = 1$, $z = 0$. 4+8+2+3+3

3. (a) Find the work done in moving a particle in a force field given by

$\vec{F} = (2x - y + z)\hat{i} + (x + y - z)\hat{j} + (3x - 2y - 5z)\hat{k}$ along a curve $x^2 + y^2 = 9$, $z = 0$.

(b) State Green's theorem. Use Green's theorem to evaluate

(i) $\oint_\Gamma \left(x^2 dx + xydy\right)$ where Γ is a square in x-y plane with vertices given by (0,0, (a,0) (a,a) and (0,a) described in positive sense.

(ii) $\oint_\Gamma \left\{\left(\cos x \sin y - xy\right)dx + \sin x \cos y\, dy\right\}$ where Γ is a circle in x-y plane given by $x^2 + y^2 = 1$ described in positive sense. 5+2+7+6

(b) In any triangle ABC, with usual notations prove that

(i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(ii) c = a cos B + b cos A

(c) Find the value of the constant d such that the vectors $\left(2\hat{i} - j + k\right), \left(\hat{i} + 2j - 3k\right)$ and $\left(3\hat{i} + d\hat{j} + 5\hat{k}\right)$ are coplanar.

6+8+6

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(b) Examine the convergence of the following series :

(i) $\frac{1+2}{2^3}+\frac{1+2+3}{3^3}+\frac{1+2+3+4}{4^3}+\ldots\ldots..$

(ii) $1+\frac{1}{1!}+\frac{2^2}{2!}+\frac{3^3}{3!}+\ldots\ldots.$

(iii) $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\left(\frac{4}{9}\right)^4+\ldots\ldots.$ 8+4+4+4

7. (a) Two sequences $\{x_n\}$, $\{y_n\}$ are defined by $x_{n+1}=\frac{1}{2}(x_n+y_n), y_{n+1}=\sqrt{x_n y_n}$ $for\ n\geq 1$ and $x_1>0$, $y_1>0$. Prove that both the sequences converges to a common limit.

(b) Test the convergence of the series $\sum_{n=1}^{\infty}u_n$ where $u_n=\sqrt[3]{n^3+1}-n$.

(c) Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$. 8+6+6

8. (a) If the volume of tetrahedron be 2 units and 3 of its vertices A(1,1,0), B(1,0,1), C(2,–1,1). Then find the locus of fourth vertex.

4. (a) Find directional derivative of $f=xy+yz+zx$ in the direction of vector $\hat{i}+2\hat{j}+2\hat{k}$ at (1,2,0).

(b) Determine the constant 'a' such that $\vec{V}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal.

(c) Find a, b, c such that

$\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational.

(d) Prove that

$$\nabla\left(\vec{F}.\vec{G}\right)=\vec{F}\times curl\vec{G}+\vec{G}\times curl\,\vec{F}+\left(\vec{F}.\nabla\right)\vec{G}+\left(\vec{G}.\nabla\right)\vec{F}$$

5+4+5+6

5. (a) Prove that $\lim_{n\to\infty} r^n=0$ if $|r|<1$

(b) Prove that the sequence $\{u_n\}$ defined by $u_1=\sqrt{2}$ and $u_{n+1}=\sqrt{2u_n}$ for all $n\geq 1$ converges to 2.

(c) Define Cauchy Sequence. Prove that the sequence $\{(-1)^n\}$ is not a Cauchy Sequence. 6+7+7

6. (a) Prove that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for $p>1$ and diverges for $p\leq 1$.