Ref. No. EX/PRN/MATH/T/112/2019(Old)

Bachelor of Printing Engineering Examination (Old), 2019

(1st Year, 1st Semester)

MATHEMATICS IR

Full Marks:100

Time: Three hours

Symbols/Notations have their usual meanings

Answer **any FIVE** questions

(1). (a) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

(b) Is Rolle's theorem applicable to $f(x) = 4 + (6-x)^{(2/3)}$? Justify your answer.

(c) Find n-th derivative of $y = \sin(ax+b)$.

6 + 8 + 6

(2). (a) If $y = (sin^{-1}x)^2$, Prove that,

(i) $y_2(1-x^2) - xy_1 - 2 = 0$

(ii)$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$

(c) Show that the maximum value of $x^{\frac{1}{x}}$ for $x > 0$ is $e^{\frac{1}{e}}$.

6 + 6 + 8

(3). (a) Prove that $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$.

(b) State Lagrange's Mean Value theorem. Use this theorem to prove that $\frac{x}{1+x} < \ln(1+x) < x$ for $x > 0$.

(c) Verify Lagrange's Mean Value theorem for the function

$$f(x) = x(x-1)(x-2), 0 \leq x \leq \frac{1}{2}.$$

7 + 7 + 6

(4). (a) Find the value of a so that $\lim_{x\to 0}\frac{e^{ax} - e^x - x}{x^2}$ exists finitely.

(b) Examine the convergence of the integral $\int_2^\infty \frac{x^3}{\sqrt{x^7+1}}\, dx$.

(c) Find the maximum and minimum of

$$f(x) = \sin x(1+\cos x)$$

on the interval $[0, 2\pi]$.

8 + 6 + 6

(5). (a) Find the maximum, minimum values (if exist) of $f(x,y) = x^3 + y^3 + 3xy$.

(b) Find the area bounded by the rectangular hyperbola $xy = c^2$, x axis and the ordinates $x = c$, $x = 2c$.

(c) Find the volume of a solid generated by the revolving about x-axis the area under one arch of cycloid $x = \theta - \sin\ \theta$, $y = 1 - \cos\ \theta$.

7 + 6 + 7

(6). (a) Examine whether i) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x+y}$, ii) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2y}{x^2+y^2}$ exists. Justify your answer.

(b) If $u = tan^{-1}r$, $v = tan^{-1}x/r$, $r^2 = x^2 + y^2$, then show that $yu_x = xu_y$ and $xv_x + yv_y = 0$

(c) If $u = F(p)G(q)$ where $p = x^2 + y^2 + z^2$, $q = xy + yz + zx$, then find the value of $(y - z)u_x + (z - x)u_y + (x - y)u_z$.

6 + 8 + 6

(7). Find the value of $x^2\, u_{xx} + 2xy\, u_{xy} + y^2\, u_{yy}$ if

(i) $u = cos^{-1}\, \frac{x+y}{x^{1/2}+y^{1/2}}$,

(ii) $u = tan^{-1}\, \frac{x^3+y^3}{x-y}$,

10 + 10

(8). (a) Show that the area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$.

(b) Evaluate $\int_0^1 (x^2 + 13)\, dx$ by Trapezoidal rule and Simpson's one third rule.

6 + 7 + 7