Ex/IT/T/424/2018

JADAVPUR UNIVERSITY

B.E. INFORMATION TECHNOLOGY

4th Year, 2nd Semester Examination - 2018

DIGITAL SIGNAL PROCESSING **Time : 3 hours** **Full Marks : 100**

General instructions (read carefully)

1. Special credit will be given to answers which are brief and to the point.
2. Answer to every question should start on a new page.
3. Do not write answers to various parts of a question at different locations of your answer-script.
4. Do not write on the front back cover of your answer booklet.

Marks for each sub-part of a question is mentioned at the right margin of a part question or set of part questions.

Part 1 (any one question to be answered)

1. i) What are the advantages of Digital Signal Processing over Analog (any 6 points)? (3)

ii) The output y(n) and the input x(n) of a discrete time system are related by the equation $y(n) = e^{x(n)}$. Determine whether the system is linear, time - invariant and stable. (7)

2. i) What are causal and non-causal signals. Illustrate with examples. (3)

ii) Define LTI system with an example. (3)

iii) The impulse response of an LTI system is $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$. Determine the response of the system to the input signal $\{\underset{\uparrow}{1}, 2, 3, 1\}$. (4)

Part 2 (any one question to be answered)

3. i) Find the Fourier Transform (FT) of the signal

$$x(n) = 2^{n} u(-n) - 2^{-n} u(n) \quad (5)$$

ii) Find out the Discrete Fourier Transform (DFT) of the sequence, using matrix representation

$$x(n) = \{2, 1, 2, 1\} \quad (5)$$

iii) Find the Inverse DFT (IDFT) of

$X(k) = \{ 6, -2 + 2j, -2, -2 - 2j \}$ (5)

iv) For calculating the DFT of a sequence x(n) of length N, compare the number of computations when done directly vis-à-vis when done using FFT algorithm.

If N = 1024, and if the computation of DFT directly takes 10 seconds, how much time will it take using FFT algorithm ? (5)

4. i) Determine the Z transform of the following sequence and find the ROC.

$x(n) = (n + 2)\,(1/2)^n\,u(n).$ (6)

ii) Find the Inverse Z transform of

$X(z) = z\,(z^2 - 4z + 5) / (z - 3)\,(z - 2)\,(z - 1), \quad 2 < z < 3.$ (6)

iii) Verify the stability of the system having impulse response

$h(n) = (1/2)^n\,u(n)$ (3)

iv) What is Parseval's relation for the energy of a sequence, using Fourier Transform, and Discrete Fourier Transform. (2 x 2.5)

Part 3 (any one question to be answered)

5. i) What is the main characteristic of a Linear Phase Filter ? Name an application where such a filter finds practical use. Such linear phase filters can be realized by which one of the following :- FIR digital filters, IIR digital filters, Analog filters. (1 + 1 + 1)

ii) Derive the frequency response $H(e^{j\omega})$, Phase (ϕ), Group Delay (τ_g), and Transfer function H(z) of **any one** of Types 1, 2, 3, or 4 Linear Phase Filters. (12)

iii) What are the restrictions, if any, of the type chosen in ii) only, for realization of LPF, HPF, BPF and BSF ? Justify your answer. (3)

iv) Which type of Linear Phase filters can be used to implement a Differentiator ? Justify your answer. (2)

6. i) What is a Delay Complementary filter ? (2)

ii) Suppose we want to construct a Delay Complementary filter for a Type 1 Linear Phase LPF having Frequency Response $H_0(e^{j\omega})$ and of odd length 2K + 1 (hence order 2K), having a tolerance band of 1 +/- δ_p in the passband and δ_s in the stopband.

What kind of filter will the Delay Complementary filter be ? What would be the tolerance band of such a filter ? Explain with the help of the filter pseudo magnitude vs frequency characteristics diagram. (8)

iii) What are Doubly Complementary filters ? Explain using two all-pass complementary transfer functions $H_0(z) = ½\,[A_0(z) + A_1(z)]$ and $H_1(z) = ½\,[A_0(z) - A_1(z)]$. (6)

iv) Explain the terms Cross-over frequency and Cross-over Networks. In what application such networks find widespread use ? (4)

Part 4 (any one question to be answered)

7. i) Explain Aliasing distortion with a concrete example. (4)

ii) What is Sampling Theorem? (2)

iii) Explain the terms Nyquist Frequency and Baseband. (4)

8. i) What is the difference between Type 1 and Type 2 Chebyshev filters ? (2)

ii) For a Type 1 Chebyshev LPF, draw and explain the frequency response characteristics, $H_a(j\Omega)$ vs Ω, for values of the order of the filter, N = 1, 2, 3, 4. (8)

Part 5 (any two questions to be answered)

9. i) For a Band-Pass IIR digital filter, write and explain

   a. Transfer function, $H(z)$
   b. Frequency response characteristics, $|H(e^{j\omega})|$ vs ω
   c. Central frequency ω_0,
   d. Bandwidth
   e. Q of the filter.

(12)

ii) Determine the location of the zeros and poles (in the Z plane) for the Band - Pass IIR digital filter, with $\omega_0 = \pi / 2$ and Bandwidth = $\pi / 4$. (5)

iii) Where would the zeros and poles be for a Band - Stop IIR digital filter, with the same parameters as in ii). Deduce after writing the Transfer function. (3)

10.

i) What is Delay Equalization ? Illustrate using Phase-Frequency characteristic curve. (3)

ii) Write about three important properties of an All Pass digital filter. (9)

iii) Explain the functioning of a Comb filter, taking a LPF Transfer function, $H(z^L)$, with L number of delays. Draw the frequency response characteristics of such a comb filter taking L = 5. (6)

iv) Write about one common application of such a filter, mentioned in iii). (2)

11. i) Derive mathematically the expression for the Asymptotic slope in case of a Butterworth Filter. If we are required to design such a filter with an asymptotic slope of 24 dB / octave, what order of the filter do we require ? (3 + 1)

ii) Derive the Transfer function, $H_a(s)$, with calculated values of b_k, of a Butterworth Filter for

   a. $N = 4, \Omega_c \neq 1$
   b. $N = 5, \Omega_c = 1$

(3 + 3)

iii) Given the 3 dB frequency, $\Omega_c = 1000\,\pi$, $\Omega_s = 2000\,\pi$, and attenuation in the stop-band $\delta_s \geq$ 40 dB, find for a Butterworth Filter

a. The exact value of the order N.
b. The chosen value of the order, N and the corresponding value of the stop-band edge frequency Ω_s
c. The Transfer function, $H_a(s)$, with calculated values of b_k, for the designed filter.

(10)

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