EX/ IT/T/326/2018

B. INFO. TECH 3RD YEAR 2ND SEMESTER EXAMINATION, 2018

MATHEMATICAL METHODS

Time: Three hours

Full Marks: 100

Different parts of the same question should be answered together.

CO1 [20]

[1] Answer the following questions.

(a) An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword.

i) Let X be the number of websites visited until the first keyword is found. Find the distribution of X.

ii) Compute the expected value and the standard deviation of X.

iii) Out of the first 10 websites, let Y be the number of sites that contain the keyword. Find the distribution of Y.

iv) Compute the expected value and the standard deviation of Y.

v) Compute the probability that at least 5 of the first 10 websites contain the keyword.

vi) Compute the probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword.

[10]

(b) Identical computer components are shipped in boxes of 5. About 15% of components have defects. Boxes are tested in a random order.

i) What is the probability that a randomly selected box has only non-defective components?

ii) What is the probability that at least 8 of randomly selected 10 boxes have only non-defective components?

iii) What is the distribution of the number of boxes tested until a box without defective components is found?

[10]

CO2 [20]

[2] *Answer* either (a) or (b) *in this block*

(a) Answer the following questions.

(i) What is meant by k-th order statistic?

(ii) What is the probability distribution of k-th order statistic?

(iii) Which order statistic represents overall series system lifetime?

(iv) Which order statistic represents the lifetime of a parallel system?

(v) Which order statistic will represent the lifetime of an *k-out-of-n* system?

(vi) Give the formulation for Reliability of a k out of n system.

(vii) What is Triple Modular Redundancy?

(viii) What is the expression for Reliability in TMR?

[8x2.5 = 20]

(b) Consider a series connection of two components, with respective lifetimes X and Y. The joint pdf of the lifetimes is given by

$$f(x,y) = \begin{cases} \frac{1}{200}, & (x,y) \in A \\ 0, & elsewhere \end{cases}$$

where A is the triangular region in the (x, y) plane with the vertices (100, 100), (100, 120), and (120, 120). Find the reliability expression for the entire system.

[20]

CO3 [20]

[3]

(a) Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates $\lambda_1=1$ and $\lambda_2=2$ respectively. Let $N(t)$ be the merged process $N(t)=N_1(t)+N_2(t)$.

i. Find the probability that $N(1)=2$ and $N(2)=5$.

ii. Given that $N(1)=2$, find the probability that $N_1(1)=1$.

[10]

(b) Let $\{N(t), t\in[0,\infty)\}$ be a Poisson process with rate λ, and X_1 be its first arrival time. Show that if $N(t)=1$, X_1 is uniformly distributed in (0,t] That is, show that

$$P(X_1 \leq x \mid N(t)=1) = x_t, \text{ for } 0 \leq x \leq t.$$

[10]

CO4 [20]

[4]

A computer is shared by 2 users who send tasks to a computer remotely and work independently. At any minute, any connected user may disconnect with probability 0.5, and any disconnected user may connect with a new task with probability 0.2. Let $X(t)$ be the number of concurrent users at time t (minutes). This is a Markov chain with 3 states: 0, 1, and 2.

Compute transition probabilities and transition diagram.

If both users are connected at 10:00, what is the probability that there will be no users at 10:02?

[20]

CO5 [20]

<u>[5] *Answer any* two out of (a), (b) and (c) from this block:</u>

(a) A system is being designed. The inter-arrival times of customers are expected to be exponentially distributed with mean $1/\lambda = 50$ msec. Three options are considered.

(i) One single-server queue with infinite buffer space. The service times are exponentially distributed with mean $1/\mu = 20$ msec.

(ii) Two single-server queues, each with infinite buffer space. Customers are randomly dispatched to each queue with an equal probability. The service times are exponentially distributed with mean $1/\mu = 40$ msec at each server.

(ii) One two-server queue with infinite buffer space. The service times are exponentially distributed with mean $1/\mu = 40$ msec at each server.

Find the response time in each option using queuing analysis.

[10]

(b) In a health clinic, the average rate of arrival of patients is 12 patients per hour. On an average, a doctor can serve patients at the rate of one patient every four minutes.

Assume, the arrival of patients follows a Poisson distribution and service to patients follows an exponential distribution.

(i) Find the average number of patients in the waiting line and in the clinic.

(ii) Find the average waiting time in the waiting line or in the queue and also the average waiting time in the clinic.

[10]

(c) Consider an *M/M/*1 queuing system in which the total number of jobs is limited to *n* owing to a limitation on queue size.

(i) Find the steady state probability that an arriving request is rejected because the queue is full.

(ii) Find the steady-state probability that the processor is idle.

(iii) Find the throughput of the system in the steady state.

(iv) Given that a request has been accepted, find its average response time.

[10]