EX/ IT/MATH/T/212/2018(S)

BACHELOR OF INFORMATION TECHNOLOGY ENGG.
SUPPLEMENTARY EXAMINATION - 2018
(2[ND] YR. 1[ST] SEM.)
MATHEMATICS-III(Modules 3 & 10)

Time: Three hours   Full Marks: 100

Answer any **Ten** questions 10×10

1. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A - 5I_3 = 0$, Hence obtain a matrix B such that $AB = I_3$. 10

2. Find the value of $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & -e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$, using Laplace's method. 10

3. (i) Expresss $A = \begin{pmatrix} 1 & 3 & 4 \\ 7 & 2 & 6 \\ 2 & 8 & 1 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

   (ii)If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, show that $A^2 - 2A + I_2 = 0$, Hence find A^{50}. 5+5

4. Solve by Cramer's rule $x + 2y - 3z = 1; 2x - y + z = 4; x + 3y = 5$. 10

5. Reduce the matrix $A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \\ 4 & -4 & 8 & 9 \end{pmatrix}$ to the normal form and find its rank. 10

6. Investigate for what value of α and β the following equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \alpha z = \beta$ have (i)no solution; (ii)a unique solution and (iii)an infinite number of solutions. 10

7. Statement of Cayley Hamilton theorem. Verify Cayley Hamilton theorem for the matrix A. Express A^{-1} as a polynomial in A and then compute A^{-1}, where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. 10

8. Find the eigen values and eigen vectors of the given matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. 10

9. Próve that the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represent a pair of planes, if $+2fgh - af^2 - bg^2 - ch^2 = 0$, and also prove that the angle between the planes is $\tan^{-1} \frac{2(f^2+g^2+h^2-bc-ca-ab)^{\frac{1}{2}}}{a+b+c}$. 10

10. Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0, z + x = 0, x + y = 0, x + y + z = -c$ is $\frac{2c}{\sqrt{6}}$ and the three lines of the S. D. Intersect at the point $= y = z = -c$. 10

11. Show that the equation of the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$ and if $2d$ be the shortest distance, then find the value of $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} =?$ 10

12. Prove that the set $G = \{(\cos\theta + i\sin\theta): \theta\ runs\ over\ rational\ number\}$ forms an infinite abelian group with respect to ordinary multiplication. 10

13. Let $M = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : where\ a, b, c, d\ are\ real\ numbers\ and\ ad - bc \neq 0\}$. Prove that Mbecomes a non-cmmutative group under usual matrix multiplication. 10