EX/ IT/MATH/T/212/2018

BACHOLOR OF INFORMATION TECHNOLOGY ENGG.
EXAMINATION - 2018
(2^{ND} YR. 1^{ST} SEM.)
MATHEMATICS-III(Modules 3 & 10)

Time: Three hours Full Marks: 100

Answer any **Ten** questions 10×10

1. (a) (i) Express $A = \begin{pmatrix} 1 & 3 & 4 \\ 7 & 2 & 6 \\ 2 & 8 & 1 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

   (ii)If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, show that $A^2 - 2A + I_2 = 0$, Hence find A^{50}. 3+2

   (b)Find the value of $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$, using Laplace's method. 5

2. (i)Express as the product of two determinants and hence prove that $\begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix} = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$

   (ii)Solve by Cramer's rule $x - y + z = 0; 2x + 3y - 5z = 7; 3x - 4y - 2z = -1$. 5+5

3. (a)Obtain the fully reduced normal form of the matrix $A = \begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$ and find its rank.

   (b)Show that the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix}$ is non-singular and express it as a product of elementary matrices. 6+4

4. Statement of Cayley Hamilton theorem. Verify Cayley Hamilton theorem for the matrix A. Express A^{-1} as a polynomial in A and then compute A^{-1}, where $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$. 10

5. Diagonalise the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. 10

6. A line is drawn to meet $y = x\tan\alpha$; $y = -x\tan\alpha$, $z = -c$ so that the length intercepted on it is constant. Show that $\frac{x - k\sin\theta\cot\alpha}{k\cos\theta} = \frac{y - k\cos\theta\tan\alpha}{k\sin\theta} = \frac{z}{c}$ where k is a constant and θ is a parameter. Deduce the equation to the locus of the line. 10

7. (a) Show that the locus of a variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$is the surface $y^2 - m^2x^2 = z^2 - c^2$.

(b) A variable plane at a constant distance P from the origin meets the axes at A,B,C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$. 5+5

8. (a)Show that the straight lines whose d.cs. are given by $al + bm + cn = 0, fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ *and parallel if* $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$.

(b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0, x - 2y + 3z + 1 = 0$ is a great circle. 5+5

9. Prove that the line of shortest distance between the Z-axis and the variable line $\frac{x}{a} + \frac{z}{c} = \alpha\left(1 + \frac{y}{b}\right), \frac{x}{a} - \frac{z}{c} = \frac{1}{\alpha}\left(1 - \frac{y}{b}\right)$. 10

10. Prove that the set $G = \{(\cos\theta + i\sin\theta) : \theta \text{ runs over rational number}\}$ forms an infinite abelian group with respect to ordinary multiplication. 10

11. Find the shortest distance between the lines $3x - 2y - 3z + 4 = 0 = x + 2y + z - 6,\ x + y - z = 0 = x - 2y + z$ and obtain the equations of the line along which the above shortest distance measured. 10

12. Show that all the roots of the equation $x^6 = 1$ from a cyclic group of order 6, under usual multiplication of complex numbers. 10

13. If $(R, +, .)$be a ring such that $a^2 = a$, for all $a \in R$, then prove that

(i)$a + a = 0$ *for all* $a \in R$

(ii)$a + b = 0 \Rightarrow a = b$ *for all* $a, b \in R$.

(iii)$a.b = b.a$ *for all* $a, b \in R$ 10