

BACHELOR OF INFORMATION TECHNOLOGY ENGG.  
EXAMINATION - 2018  
(2<sup>ND</sup> YR. 2<sup>ND</sup> SEM.)  
MATHEMATICS-IV(MODULES 8 & 12)

Time: Three hours

Full Marks: 100

Answer all questions attempted of (Group-A,B,C) and answer any two questions of Group-D

**Group-A(40)**

1. N letter to each of which corresponds an envelope, are placed in the envelopes at random. What is the probability that (i) no letter is palced in the right envelope? (ii) exactly r letters are placed in the right envelopes? 10
2. The joint probability density function of the random variable X, Y is  

$$f(x, y) = k(3x^2 + y^2 + 2y + 1), \text{ when } 1 \leq x \leq 3, 0 \leq y \leq 2$$

$$0, \text{ elsewhere}$$

Find (i) the value of k, (ii)  $P(X+Y < 2)$ , (iii) the marginal distribution of X & Y.Investigate whether X & Y are independent. 10

3. (i) If the probability density function of a random variables X is given by  $f(x) = ce^{-(x^2+2x+3)}$ ,  $-\infty < x < \infty$  find the value of c, the expectation & variance of the distribution.
- (ii) If a coin is tossed repeatedly, show that the probability of getting m heads before n tails is  $\frac{1}{2^{m+n-1}} \sum_{i=m}^{m+n-1} m + n - 1 c_i$  7+3
4. (i) A random variable X has a density function  $f(x)$  is given by  

$$f(x) = e^{-x}, x \geq 0$$

$$0, \text{ elsewhere}$$
show that Tchebycheff's inequality gives  $P(|X - 1| \geq 2) \leq \frac{1}{4}$  and show that actual probability is  $e^{-3}$ . 5+5
- (ii) out of two regression lines given by  $x + 2y = 5$  and  $2x + 3y = 8$ , which one is the regression line of x on y? Find also the values of  $\bar{x}$ ,  $\bar{y}$ , r and  $\sigma_y$ , given  $\sigma_x = 12$ .

**Group-B(20)**

5. (i) Show that  $D_{70} = \{1, 2, 5, 7, 10, 14, 35, 70\}$  is Boolean algebra where +, . and - are defined by  $x + y = \text{lcm}\{x, y\}$ ,  $x \cdot y = \text{gcd}\{x, y\}$  and  $\bar{x} = \frac{70}{x}$  and 1 is the zero element and 70 is the one element.
- (ii) Find d.n.f and c.n.f of  $(x, y, z) = \overline{\overline{x+y} + (\bar{x}z)}$ . 10
6. Draw the logic circuit L with inputs a, b, c and output B which corresponds to each Boolean expression (i)  $B = a\bar{b}c + a\bar{c} + \bar{b}c$  (ii)  $B = \bar{a}bc + \bar{a}b\bar{c} + ab\bar{c}$ . 10

**Group-C(20)**

7. If  $n$  be a positive integer  $\geq 2$  and  $a$  be a positive real number, show that there exists a unique positive real number  $x$  such that  $x^n = a$ . 10
8. Let  $S = \left\{(-1)^m + \frac{1}{n} : m \in N, n \in N\right\}$  (i) show that 1 and -1 are limit points of  $S$   
(ii) Find  $S'$  (the derived set of  $S$ ). 10

**Group-D(20)**

9. From the following data, obtain the line of regression of  $X$  on  $Y$  10

sales $X$	91	97	108	121	67	124	51	73	111	53
purchases $Y$	71	75	69	97	70	91	39	61	80	47

10. The joint probability density function of the random variable  $x$  &  $y$  is  
 $f(x, y) = k(1 - x - y)$ , for  $x \geq 0, y \geq 0, x + y \leq 1$   
 0, elsewhere, where  $k$  is a constant. Find (i) the mean value of  $y$  when  $x = \frac{1}{2}$ . (ii) the covariance of  $x$  and  $y$ . 10
11. (a) The random variable  $X$  has the distribution given by  $P(X = k) = 2^{-k}, k = 1, 2, \dots$   
 find the value of  $E(X)$  &  $Var(X)$ .  
 (b) Find the variance for the continuous random variable  $X$  with probability density function  $f(x) = 1 - |1 - x|, 0 < x \leq 2$   
 0, elsewhere 10
12. Show that there exists a unique positive real number  $x$  such that  $x^2 = 2$ . 10