

Time : Three Hours

Full Marks : 100

Answer Part - A (compulsory) (10x2=20)

And any Five from Part - B (5x16=80).

Part - A (Compulsory) (10x2=20)

1.(a) State and prove Varignon's theorem.

(b) Prove that $\text{Cos}(\vec{A}, \vec{B}) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \cos \gamma$ where α, β, γ are the direction cosines of the vectors \vec{A} and \vec{B} respectively.

(c) A simple supported beam is loaded as shown in Fig.1. Find the reactions at A.

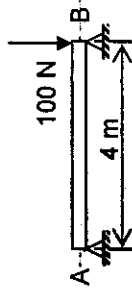


Fig.1

(d) Show with diagram the nature of reactive forces for the following cases.

i) Smooth hinge joint ii) Smooth knife edge.

(e) The coefficient of static friction μ_s between the 100 kg body and the 15° wedge is 0.2. Draw FBD of the block and wedge separately when the tendency of block is to move lower the 100 kg body if rollers of negligible friction are present under the wedge, as in Fig. 2.

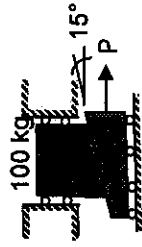


Fig.2

(f) A particle moves along a circular path of radius 1m with a uniform velocity of 2m/sec. Find its acceleration.

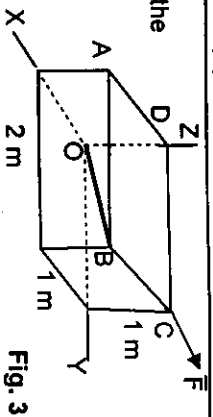
(g) Draw the graph of stress-strain for a mild steel specimen indicating all types of stresses.

(h) The position vector of a point at time t is $r = a \cos \omega t \hat{i} + b \sin \omega t \hat{j}$. Find out the equation of its path.

(i) The velocity of a particle of mass 4 kg is $(5\hat{i} + 3\hat{j})$ m/s when it is at the point (2, 1)m. Find out its angular momentum.

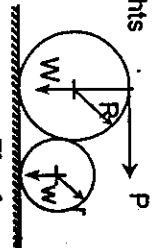
(j) Prove that tangent of angle of friction will be equal to the coefficient of friction.

2. (a) Determine the moment produced by the force $\vec{F} = (-6\hat{i} + 3\hat{j} + 10\hat{k})$ N about the diagonal OB of the rectangular block as shown in Fig. 3. Express the result as a Cartesian vector.

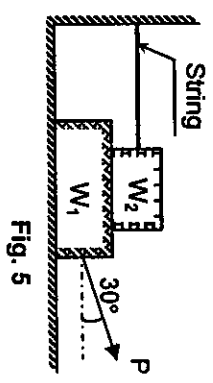


- (b) The line of action of a 50 N force passes through points A(1, 8, 7) and B(7, 1.6, 2.2). The co-ordinates are given in meters. What is the moment of this force about a point C(4, 6, 3)?

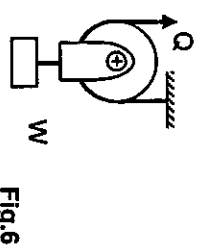
3. (a) Two heavy circular cylinders of radii R and r and weights W and w respectively rest on a horizontal plane as shown in Fig. 4. The coefficient of friction for all contact surfaces is μ . A horizontal force P is applied to pull the larger cylinder over the smaller one without sliding or rolling the smaller cylinder. Draw FBD of the larger cylinder and find the relation between R, μ , and r.



- (b) A block of weight $W_1 = 100$ N rests on a horizontal surface and supports on top of it another block of weight $W_2 = 50$ N as in Fig. 5. Find the magnitude of the horizontal force P applied to the lower block for its impending motion. The coefficient of friction for all surfaces are 0.4.



4. (a) Determine the tension Q in the cable to raise the load W = 1000 N as shown in Fig. 6. Diameter of the pulley is 400 mm and the coefficient of friction for the 40 mm bearing is 0.22. The mass of the cable and pulley is small and may be neglected.



- (b) A river is flowing with a velocity of 10 m/s from the north to south at latitude of 60° N. Determine the coriolis component of acceleration assuming the radius of earth to be 6.37×10^6 m.

5. (a) A clutch is required to transmit 10 hp at 2400 rpm. The clutch is single plate type having both sides effective. The coefficient of friction may be as 0.3 and the axial pressure is limited to 0.5 kg/cm^2 of plate area. The outside diameter of the clutch is 1.5 times the inside diameter. Assume uniform wear and determine the inside and outside diameters of the plates.
- (b) An electric train enters a curve of radius 600 meter with a speed of 30 and leaves the bend with a speed of 40 kmph during which it covers a distance of 200 m. Determine the total acceleration when it leaves the curve.

6. (a) A long jumper approaches his take-off board A with a horizontal velocity 10 m/s. Determine the vertical component of the velocity of his center of gravity at take-off for him to make the jump. What is the vertical rise of center of gravity?
- (b) A 900 N man lowers himself with the rope over a horizontal limb of a tree as shown in Fig. 7. If $\mu = 0.42$, find the force which the man must exert on the rope to let himself down slowly.



7. (a) A particle undergoing rectilinear motion along the x-axis starts at the origin with an acceleration of $10 - 3x^2 \text{ m/sec}^2$. Determine the displacement when the velocity will be again zero and when the velocity will be maximum.
- (b) Calculate the moment of inertia of a circular cylinder of radius R, length l and weight density ρ , about its axis.

8. (a) A cantilever beam of length 2 m is loaded at the free end by 2 N. Draw shear force and bending moment diagram.

- (b) A brass bar having a cross-sectional area of 1000 mm^2 is subjected to axial force shown in Fig. 8. Draw FBD of each part and find the total change in length of the bar. Take $E = 10.5 \times 10^5 \text{ N/mm}^2$

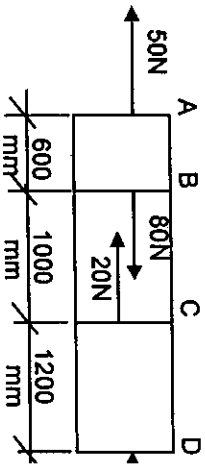


Fig. 8.