

## BACHELOR OF INFORMATION TECHNOLOGY ENGG.

EXAMINATION - 2018

(1<sup>ST</sup> YR. 2<sup>ND</sup> SEM.)

## MATHEMATICS-II(MODULES 5 &amp;6)

Time: Three hours

Full Marks: 100

## Group-A(40)

1. Represent  $f(x)$ , where  $f(x) = \cos kx$ , on  $-\pi \leq x \leq \pi$  ( $k$  not being an integer) in Fourier series. Hence deduce that (i)  $\pi \cot k\pi = \frac{1}{k} + 2k \sum_{n=1}^{\infty} \frac{1}{k^2 - n^2}$   
(ii)  $\frac{\pi}{\sin k\pi} = \sum_{n=0}^{\infty} (-1)^n \left\{ \frac{1}{n+k} + \frac{1}{n+1-k} \right\}$ . 10
2. Find Fourier sine transform of the function  $f(x) = \frac{1}{x(x^2+a^2)}$ . 10
3. (i) Show that  $L \left\{ \frac{\sin t}{t} \right\} = \tan^{-1} \frac{1}{p}$  and hence find  $\left\{ \frac{\sin at}{t} \right\}$ . Dose the Laplace transform of  $\frac{\cos at}{t}$  exists?  
(ii) Obtain the Laplace transform of  $t^2 e^t \sin 4t$  6+4
4. Solve the difference equation  $f(n+2) - 4f(n+1) + 4f(n) = 2^n$ , with  $f(0) = 1, f(1) = -1$ . 10

## Group-B(30)

5. (i) Solve  $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$   
(ii) Solve  $\frac{dy}{dx} + y \cos x = y^n \cot x$  5+5
6. (i) Solve  $(D^2 - 4D + 4)y = 8x^4 e^{2x} \sin 4x$   
(ii) Solve  $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0$  7+3
7. Solve the equation  $\frac{d^2y}{dx^2} + (x-1)^2 \frac{dy}{dx} - 4(x-1)y = 0$  in series about the ordinary point  $x = 1$ . 10

## Group-C(10)

8. (i) Solve  $(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$   
(ii) Find the complete integral of the equation  $z^2(p^2 + q^2) = x^2 + y^2$ . 5+5

## Answer any two questions of Group-D(20)

9. Find the Fourier transform of  $f(x)$ , where  $f(x) = 1 - x^2, |x| < 1$   
 $0, |x| > 1$   
and deduce the value of  $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$ . 10
10. Find the Fourier series of the function  $f(x)$  defined by  $f(x) = x^2, -\pi \leq x \leq \pi$ .  
Hence show that, (i)  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \dots = \frac{\pi^2}{12}$ ; (ii)  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{6}$ ; (iii)  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \dots = \frac{\pi^2}{8}$  10
11. Find the value of  $L^{-1} \left\{ \frac{2p+1}{(p+1)^2(p^2+1)(p^2+p+1)} \right\}$ . 10
12. (a) Solve  $(x^2 D^2 + xD - 1)y = x^2 e^{2x}$   
(b) Solve by method of variation of parameters, the equation  $(D^2 - 1)y = \frac{2}{1+e^x}$ . 5+5