EX/ IT/MATH/T/114/2018(S)

BACHELOR OF INFORMATION TECHNOLOGY ENGG.
SUPPLEMENTARY EXAMINATION - 2018
(1ST YR. 1STSEM.)
MATHEMATICS-I-(MODULE I & II)

Time: Three hours Full Marks: 100

Answer any **Ten** questions 10×10

1. State Sandwich theorem. Prove that the sequence $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots\ldots+\frac{1}{\sqrt{n^2+n}}\right)=1$, using Sandwich theorem. 10
2. Prove that the sequence $\{u_n\}$ defined by $u_1=\sqrt{7}$ and $u_{n+1}=\sqrt{(7+u_n)}$ for all $n\geq 1$ converges to the positive root of the equation $x^2-x-7=0$. 10
3. (a)Test the convergence of the series $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\cdots\ldots\ldots$
(b)Test the convergence of the series, $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+\cdots\ldots\ldots\infty$ 10
4. Prove that an infinite series $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots\ldots\ldots+\frac{1}{n^p}+\cdots\ldots\ldots$ converges if $p>1$, diverges if $p\leq 1$. 10
5. If $y=\sin(msin^{-1}x)$, then show that (i) $(1-x^2)y_2-xy_1+m^2y=0$; (ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$, 10
6. If $y=x^{n-1}\log x$, then show that $y_n=\frac{(n-1)!}{x}$ 10
7. (a)If $f(x)=(x-a)^m(x-b)^n$ where m and n are positive integers, show that c in Rolle's theorem divides the segment $a\leq x\leq b$ in the ratio $m:n$
(b)Use MVT, show that $\frac{\tan x}{x}>\frac{x}{\sin x}$ when $0<x<\frac{\pi}{2}$. 10
8. If $f(h)=f(0)+hf'(0)+\frac{h^2}{2!}f''(0)+\frac{h^3}{3!}f'''(\theta h), 0<\theta<1$ find θ when $h=8, f(x)=\frac{1}{1+x}$ 10
9. Find the value of a, b and c such that $\lim_{x\to o}\frac{x(a+b-\cos x)-c\sin x}{x^5}=1$ 10
10. If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters on an ellipse, then find the value of $\rho_1^{\frac{2}{3}}+\rho_2^{\frac{2}{3}}$ 10
11. (i) $\int_0^{\pi}\frac{x\,dx}{(a^2\cos^2x+b^2\sin^2x)^2}$ 6
(ii)$\int\frac{dx}{(x^3+1)(x+2)}$ 4
12. If $u=\log(x^3+y^3+z^3-3xyz)$, then find the value of (i) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$
(ii) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u$ and (iii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}$ 10
13. If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$, find the value of (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ and (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ 10