EX/ IT/MATH/T/114/2018

BACHOLOR OF INFORMATION TECHNOLOGY ENGG. EXAMINATION - 2018
(1ST YR. 1STSEM.)
MATHEMATICS-I-(MODULE I & II)

Time: Three hours Full Marks: 100

Answer any **Ten** questions 10 × 10

1. Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{7}$ and $u_{n+1} = \sqrt{(7+u_n)}$ for all $n \geq 1$ converges to the positive root of the equation $x^2 - x - 7 = 0$. 10
2. (a)Test the convergence of the series $\sum \frac{4.7.........(3n+1)}{1.2.......n} x^n$
   (b)Test the series for convergence, if $\beta - \alpha \neq 1$
   $1 + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \frac{(1+\alpha)(2+\alpha)(3+\alpha)}{(1+\beta)(2+\beta)(3+\beta)} + \cdots \ldots\ldots\ldots\ldots\ldots$ 5+5
3. (a)if a function $f(x)$ is such that its derivatives $f'(x)$is continuous in $[a, b]$ then prove that there exists a number $c.(a < c < b)$ such that $f(b) = f(a) + (b-a)f'(a) + \frac{1}{2}(b-a)^2 f''(c)$
   (b)Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$ $for\ 0 < x < \frac{\pi}{2}$ 5+5
4. (a)Find the value of a, b and c such that $\lim_{x\to o} \frac{x(a+b-\cos x)-c\sin x}{x^5} = 1$
   (b) If $a > b > 0$ and $(\theta) = \frac{(a^2-b^2)\cos\theta}{a-b\sin\theta}$, then find the maximum value of $f(\theta)$.5+5
5. If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters on an ellipse, then find the value of $\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}}$ 10
6. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find the value of (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$
   (ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u$ and (iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ 10
7. (a)Given the function $f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$, (x, y) $\neq$ (0,0)
   0 , (x, y) = (0,0)
   Find from definition, $f_{xy}(0,0) = ?\ and\ f_{yx}(0,0) =?$ verify whether $f_{xy}(0,0) = f_{yx}(0,0)$.
   (b)Differentiate $(\log x)^{\tan x}$ with respect to $\sin(m\cos^{-1}x)$. 6+4
8. If $u = \sin^{-1}\left\{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$ show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$ 10
9. (a) Prove that $\Gamma m \Gamma(1-m) = \pi\ cosec\ m\pi, 0 < m < 1$
   (b) Find the value of $\lim_{x\to 0} \frac{x}{1-e^{x^2}} \int_0^x e^{t^2}\, dt$. 7+3

10. (a) Assuming the integral to be convergent, show that $\int_0^{\frac{\pi}{2}} \log \sin x \ dx = \frac{\pi}{2} \log \frac{1}{2}$.

(b)Applying Beta and Gamma function, find the value of $\int_0^1 x^4 \ (1 - x^2)^3 \ dx$. 6+4

11. Calculate by Simpson's one –third rule, the value of the integral $\int_0^1 \frac{x}{1+x} dx$ correct upto three significant figures, by taking six intervals, also find its error. 10

12. (i) $\int_0^{\pi} \frac{x \, dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^2}$ 6

(ii) $\int \frac{dx}{(x^3+1)\,(x+2)}$ 4

13. (a)If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(\theta h), 0 < \theta < 1$ find θ, when $h = 8, f(x) = \frac{1}{1+x}$.

(b)Prove that the sequence $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \frac{1}{\sqrt{n^2+3}} + \cdots \ldots + \frac{1}{\sqrt{n^2+n}}\right) = 1$, using Sandwich theorem. 5+5